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**Bill Cuckler\*** ([cuckler@math.udel.edu](mailto:cuckler@math.udel.edu)), University of Delaware, Department of Mathematical Sciences, Ewing Hall, Newark, DE 19716, and **Jeff Kahn**. *Entropy bounds for perfect matchings and Hamiltonian cycles.*

For a graph  $G = (V, E)$  and  $\mathbf{x} : E \rightarrow \mathfrak{R}^+$  satisfying  $\sum_{e \ni v} \mathbf{x}_e = 1$  for each  $v \in V$ , set  $h(\mathbf{x}) = \sum_e \mathbf{x}_e \log(1/\mathbf{x}_e)$  (with  $\log = \log_2$ ). We show that for any  $n$ -vertex  $G$ , random (not necessarily uniform) perfect matching  $\mathbf{f}$  satisfying a mild technical condition, and  $\mathbf{x}_e = \Pr(e \in \mathbf{f})$ ,

$$H(\mathbf{f}) < h(\mathbf{x}) - \frac{n}{2} \log e + o(n)$$

(where  $H$  is binary entropy). This implies a similar bound for random Hamiltonian cycles.

Specializing these bounds completes a proof of a quite precise determination of the numbers of perfect matchings and Hamiltonian cycles in Dirac graphs (graphs with minimum degree at least  $n/2$ ) in terms of  $h(G) := \max \sum_e \mathbf{x}_e \log(1/\mathbf{x}_e)$  (the maximum over  $\mathbf{x}$  as above). For instance, for the number,  $\Psi(G)$ , of Hamiltonian cycles in such a  $G$ , we have

$$\Psi(G) = \exp_2[2h(G) - n \log e - o(n)].$$

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