NY 10025. Ramanujan Primes and Bertrand's Postulate.
The $n$th Ramanujan prime is the smallest natural number $R_{n}$ such that if $x \geq R_{n}$, then there are at least $n$ primes in the interval ( $x / 2, x]$. Bertrand's postulate is $R_{1}=2$. Ramanujan proved that $R_{n}$ exists and gave the first five values as 2,11 , 17, 29, 41. In this talk, we prove that $2 n \log 2 n<R_{n}<4 n \log 4 n$ for all $n$, and that $R_{n}$ is asymptotic to the $2 n$th prime. We also estimate the length of the longest string of consecutive Ramanujan primes among the first $n$ primes, explain why there exist more twin Ramanujan primes than expected, and make three conjectures. Our paper is to appear in the Monthly. (Received February 02, 2009)

