

1051-11-7

Hung-ping Tsao* (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. *Second generation Stirling numbers.*

For a sequence Q in a commutative ring, we define the Stirling number $s(n,k;Q)$ of the first kind with respect to Q as the sum of products of k numbers among the first n terms of Q with $s(n,0;Q)=1$ and the Stirling number $S(n,k;Q)$ of the second kind with respect to Q to be $S(n,k;Q)=s(n-k+1,1;Q)S(n,k-1;Q)-s(n-k+2,2;Q)S(n,k-2;Q)+\dots$ with the last term being $s(n,n;Q)$ for odd k and $-s(n,n;Q)$ for even k and $S(n,0;Q)=1$. We shall only consider $s(n,k;a,d)$ and $S(n,k;a,d)$ with respect to an arithmetic progression $(a+(n-1)d)$. Based on $s(n,k;a,d)=s(n-1,k;a,d)+[a+(n-1)d]s(n-1,k-1;a,d)$ and $S(n,k;a,d)=S(n-1,k;a,d)+[a+(n-k)d]S(n-1,k-1;a,d)$, we shall express Stirling numbers as linear combinations of binomial coefficients. The triangular arrays of coefficients in such linear combinations will be called second generation Stirling numbers. For example, $s(n,1;a,d)=S(n,1;a,d)=dC(n,2)+aC(n,1)$, $s(n,2;a,d)=3ddC(n+2,4)+d(3a-4d)C(n+1,3)+(a-d)(a-d)C(n,2)$ and $S(n,2;a,d)=3ddC(n+1,4)+d(3a-2d)C(n+1,3)+(a-d)(a-2d)C(n,2)$. We shall derive recursive formulas which will enable us to generate the second generation Stirling numbers of both kinds. They are the same except for the sign only when $a=d=1$. (Received March 03, 2009)