1051-53-237 **Michael Jablonski*** (mjablonski@math.ou.edu), Department of Mathematics, University of Oklahoma, Norman, OK 73019-0315. *Applications of geometric invariant theory to the geometry of Lie groups with left-invariant metrics.*

We present the basic framework for studying (simply connected) Lie groups with left-invariant metrics from the perspective of being points in a variety of Lie structures. In this setting, isomorphism classes of Lie group structures correspond to orbits of a $GL_n\mathbb{R}$-action.

The geometry of these $GL_n\mathbb{R}$-orbits has strong implications on the types of left-invariant Riemannian metrics that a group can admit. In particular, one can understand the existence of Ricci soliton (resp. Einstein) metrics on nilpotent (resp. non-unimodular solvable) groups via geometric properties of these orbits. This is the study of so-called 'distinguished orbits'.

In the nilpotent case, we show that Ricci soliton metrics are the most 'symmetric' metrics that such a group can admit; that is, they have the largest isometry groups among all left-invariant metrics. Moreover, we obtain a nice decomposition of the automorphism group of nilpotent Lie groups which admit Ricci soliton metrics. (Received August 25, 2009)