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Andre Schulz* (aschulz@email.smith.edu), Smith College, Computer Science Department, Northampton, MA 01060, and **Kevin Buchin** (buchin@cs.uu.nl). *On the Number of Spanning Trees a Planar Graph Can Have.*

The maximum number of spanning trees of a planar graph grows exponentially with the size of the graph. We prove a new upper bound for this exponential growth. The improved bound relies on a careful probabilistic analysis of the so called “outgoing edge” approach. We analyze the occurrence of directed cycles if one selects one outgoing edge for every vertex. As novelty we introduce a technique that is capable to analyze dependent events in this scenario easily. Our method might find applications on similar problems with sparse dependency graphs.

By Kirchoff’s Matrix-Tree Theorem the determinant of the Laplace matrix of a graph equals the number of its spanning trees. As a consequence the improved upper bound implies also a new upper bound for grid embedding of 3-polytopes. For this application the maximal growth is determined by planar graphs which contain no triangular and quadrilateral faces. We therefore analyze this special situation separately and obtain an improved upper bound for this setting too. (Received February 24, 2009)