Romanov proved that there is a positive proportion of odd positive integers of the form $2^{n}+p$ for some positive integer $n$ and prime $p$. Erdős proved that the odd integers not representable in this way also account for a positive proportion of all the integers. Note that if $m$ is a positive integer of the latter kind, then $m-1$ is even and not representable under the form $2^{n}+(p-1)=2^{n}+\phi(p)$ for any positive integer $n$ and prime $p$, where $\phi$ is the Euler function. Similarly, $m+1$ is even and not representable under the form $2^{n}+(p+1)=2^{n}+\sigma(p)$ for any positive integer $n$ and prime $p$, where $\sigma$ is the sum of divisors function. It makes sense to ask if we can remove the assumption that $p$ is prime in the above statements and prove that there are infinitely many positive integers not of the form $2^{n}+\phi(m)$, or not of the form $2^{n}+\sigma(m)$ for any positive integers $n$ and $m$, respectively. In my talk, I will prove that the answer to the above questions is yes. In fact, each of the above two sets of positive integers has a positive lower density. We shall also discuss some related problems and pose some open questions.
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