The Dieudonné module of the $p$-torsion group scheme is a fundamental invariant of an abelian variety in characteristic $p > 0$. There are very few cases in which the Dieudonné module of the Jacobian of a curve is known. One exception is the Hermitian curve $y^p + y = x^{p+1}$, whose Jacobian is well-known to be superspecial; equivalently, its Jacobian is isomorphic to a product of supersingular elliptic curves. For a prime power $q = p^n$ with $n \geq 2$, the Hermitian curve $H_q : y^q + y = x^{q+1}$ is no longer superspecial, but is still supersingular; equivalently, its Jacobian $\text{Jac}(H_q)$ is isogenous to a product of supersingular elliptic curves.

We determine the Dieudonné module of $\text{Jac}(H_q)$ for all prime powers $q = p^n$ using the $k[F,V]$-module structure of the de Rham cohomology of $H_q$. The indecomposable factors of the Dieudonné module have a surprising combinatorial structure; while their multiplicities depend on $p$, the structure of each indecomposable factor does not. This result has several applications: it yields constraints on the decomposition of $\text{Jac}(H_q)$ up to isomorphism; and it gives information about the intersection of the Ekedahl-Oort strata with the supersingular locus of the moduli space of abelian varieties. (Received December 12, 2011)