Christopher Davis (davis@math.uci.edu) and Kiran S. Kedlaya* (kedlaya@mit.edu). On surjectivity of the Witt vector Frobenius.

For $p$ a prime number and $R$ a ring, the $p$-typical Witt vectors over $R$ form another ring equipped with an endomorphism called Frobenius. If $R$ is of characteristic $p$, this coincides with the map induced by functoriality from the usual Frobenius map on $R$. Otherwise, the Witt vector Frobenius is somewhat mysterious; it is never injective, but it is hard to tell whether or not it is surjective. We show that surjectivity of Frobenius per se is rather rare, but that it becomes much more common if we replace infinite Witt vectors with Witt vectors of finite (but arbitrary) length. For instance, we get surjectivity in this sense if $R$ is the ring of integers in any algebraic extension of the rationals containing all roots of unity. This is closely related to such concepts in $p$-adic Hodge theory as the almost purity theorem of Faltings, which can be generalized (using work of Kedlaya-Liu and Scholze) to a statement with our surjectivity condition as the main hypothesis. One also has analogous statements for big Witt vectors, which may eventually have more global applications. (Received November 03, 2011)