Exceptional lines, separating sets and vanishing topology at singular points of complex surfaces. Preliminary report.

Let \((V, p) \subset (\mathbb{C}^3, 0)\) be a surface with isolated singularity at 0. The Nash fiber at \(p\) (which is defined as the limit of the tangent spaces to \(V\) at smooth points \(x \in V\) as \(x\) tends of \(p\)) is known to consist of the Nash fiber of the Zariski tangent cone \((C V, 0)\) (that is, all limits of tangent spaces to the reduced tangent cone) together with finitely many, possibly zero, pencils of planes whose axes are lines in \(C V\) through 0 called exceptional lines. A separating set is a three-dimensional semi-algebraic subset with real tangent cone at 0 of real dimension less than three. We show that the tangent cone to the separating set lies in an exceptional line and give examples showing that for cyclic quotient singularities, the separating sets are topologically (but not geometrically) cones over splitting tori in the link of \((V, p)\). The splitting tori figure in Birbrair, Neumann and Pichon’s thick-thin decomposition of the link. (Received December 01, 2011)