Let $G = U(2m, \mathbb{F}_q)$ be the finite unitary group defined over a finite field of order $q$, where $q$ is the power of an odd prime $p$. We prove that the number of irreducible complex characters of $G$ with degree coprime to $p$, and with Frobenius-Schur indicator $-1$, is equal to $q^{m-1}$. In particular, we find a (non-canonical) bijection between these irreducible characters and the set of self-dual polynomials of degree $2m$ over $\mathbb{F}_q$ with constant term $-1$. (Received December 07, 2011)