J. Brundan and A. Kleshchev [Advances in Math. 222 (2009), 1883-1942] showed that cyclotomic Hecke algebras can be graded, and introduced graded decomposition numbers for these algebras. Using their results, Ariki showed that $q$-Schur algebras can be graded and one can also consider graded decomposition numbers in that case. We prove similar results for $GL(n, q)$, including a graded analog of a result of Dipper-James which connects a part of the $\ell$-modular decomposition matrix of $GL(n, q)$ (where $\ell$ is prime to $q$) with the decomposition matrix of the $q$-Schur algebra. (Received November 23, 2011)