Let $F$ be a non-Archimedean local field. Bushnell and Henniart described an explicit bijection between the essentially tame Langlands parameters, i.e. irreducible representations of the Weil group $W_F$ of dimension $n$, and the essentially tame irreducible supercuspidal representations of $GL_n(F)$. This is known as the essentially tame local Langlands correspondence. We can regard each Langlands parameter as a representation twisted-induced from a character $\xi$ of a tamely ramified subgroup $W_E$ of $W_F$ of degree $n$. Here a twist refers to another character $\mu_\xi$ of $E^\times$ called the rectifier of $\xi$. We prove that the rectifier admits a factorization such that the factors are parameterized by the isotypic components of a finite symplectic module arising from the construction of the corresponding supercuspidal representation from $\xi$. With such factorization, we can express our Langlands parameter in terms of an admissible embedding of $L$-groups constructed by Langlands and Shelstad. Therefore we give a different interpretation of the essentially tame local Langlands correspondence.

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