Duality in Segal-Bargmann Spaces.

The Segal-Bargmann space is the holomorphic $L^2$ space of Gaussian measure $\gamma$ on $\mathbb{C}^n$. As a Hilbert space, it is (of course) self-dual. The corresponding holomorphic $L^p$ spaces for $p \neq 2$ have more complicated duality relations. It has been known since the mid-80s that the dual to $L^p_{hol}(\gamma)$ can be identified with $L^{p'}(\gamma_p)$ for the conjugate exponent $\frac{1}{p} + \frac{1}{p'} = 1$ and a dilated Gaussian measure $\gamma_p$; but this is not isometric.

Here we will present a tight estimate on constant of comparison between the dual norms. It grows exponentially fast with dimension $n$, leaving open many interesting questions about $L^p$ Segal-Bargmann spaces in infinite-dimensions. The key to the proof is viewing $L^p_{hol}(\gamma)$ as a space of holomorphic sections over a (in this case trivial) vector bundle; this point of view shows why the dilation of the measure is natural, and translates the problem to the $L^p$-norm of an orthogonal projection. (Received December 13, 2011)