1089-03-178 **David Chodounsky*** (david.chodounsky@matfyz.cz). *Hausdorff Gaps and Towers in* $P(\omega)/fin$.

The classical result of Kunen states that a gap $(L_\alpha, R_\alpha)_{\alpha\in\omega_1}$ in $\mathcal{P}(\omega)/fin$ is indestructible iff $(L_\alpha \cap R_\beta) \cup (L_\beta \cap R_\alpha) \neq \emptyset$ for each $\alpha < \beta \in \omega_1$. (This has to hold for some cofinal subgap.) The classical construction of Hausdorff produces a gap such that $\{\alpha < \beta \colon L_\alpha \cap R_\beta \subset n\}$ is finite for each $\beta \in \omega_1$ and $n \in \omega$. We show that this two indestructibility conditions are consistently different.

We also study combinatorics of towers - well ordered $\subset^*$-chains in $\mathcal{P}(\omega)/fin$. We call the tower $(T_\alpha)_{\alpha\in\omega_1}$ Suslin if each cofinal subtower contains two elements in inclusion, and Hausdorff if $\{\alpha < \beta \colon T_\alpha \setminus T_\beta \subset n\}$ is finite for each $\beta \in \omega_1$ and $n \in \omega$. We show that assuming MA all towers are Hausdorff, Hausdorff towers generate ideals of maximal Tukey order among posets of size ω_1 and there can be a tower, which is not equivalent to any Hausdorff nor Suslin tower.

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