

1114-11-302

**Kenneth A. Ribet\*** ([ribet@berkeley.edu](mailto:ribet@berkeley.edu)). *The Eisenstein ideal and the cuspidal group*. Preliminary report.

We present joint work with Bruce Jordan and Anthony Scholl on the Jacobian  $J$  of the modular curve  $X = X_0(N)$ , where  $N$  is a positive integer.

Let  $\tilde{\mathbf{T}}$  be the ring of Hecke operators on the space of modular forms of weight 2 for  $\Gamma_0(N)$ , and let  $\mathbf{T}$  be the image of  $\tilde{\mathbf{T}}$  in the endomorphism ring of  $J$ . The Eisenstein ideal of  $\mathbf{T}$  is the ideal of those  $t \in \mathbf{T}$  that lift to an operator  $\tilde{t} \in \tilde{\mathbf{T}}$  such that  $\tilde{t}$  vanishes on the space of Eisenstein series.

Let  $C$  be the cuspidal subgroup of  $J$ . Because we have  $I \subseteq \text{Ann}_{\mathbf{T}} C$ , it is natural to ask whether  $I = \text{Ann}_{\mathbf{T}} C$ .

We prove this equality locally at prime numbers that are prime to the product  $6N$  and expect to be able to consider more generally primes (including 2 and 3) whose squares do not divide  $N$ .

Let  $\mathcal{C}$  be the *formal cuspidal group* for  $J$ , the group of degree-0 divisors on  $X$  with support on the cusps. There is a natural map  $\mathcal{C} \rightarrow J$  whose image is  $C$ ; we regard this map as a 1-motive  $[\mathcal{C} \rightarrow J]$ . Consideration of the cohomology of this 1-motive reveals the desired connection between the Eisenstein ideal and the cuspidal group of  $J$ . (Received September 01, 2015)