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William H Kuszmaul*, william.kuszmaul@gmail.com. *Fast Algorithms for Finding Pattern Avoiders and Counting Pattern Occurrences in Permutations.*

Given a set Π of permutation patterns of length at most k , we present an algorithm for building $S_{\leq n}(\Pi)$, the set of permutations of length at most n avoiding the patterns in Π , in time $O(|S_{\leq n-1}(\Pi)| \cdot k + |S_n(\Pi)|)$. Whereas the previous best algorithms, based on generate-and-check, require exponential time per permutation analyzed, our algorithm is the first to run in time polynomial per outputted permutation. Moreover, our algorithm can be adapted to compute the cardinality of $S_{\leq n}(\Pi)$ using space $O(n^k)$.

Additionally, we present an $O(n!k)$ -time and $O(n^{k+1}k)$ -space algorithm for counting the number of copies of patterns from Π in each permutation in S_n . Surprisingly, when $|\Pi| = 1$, this runtime can be improved to $O(n!)$, spending only constant time per permutation.

Using our algorithms, we generate $|S_5(\Pi)|, \dots, |S_{16}(\Pi)|$ for each $\Pi \subseteq S_4$. For thousands of Π , we are able identify OEIS sequences which we conjecture to enumerate $S_n(\Pi)$. (Received August 23, 2016)