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James McKeown* (mckeown@math.miami.edu). *Waldspurger and Meinrenken Symmetric Group Tilings*. Preliminary report.

Consider the reflection representation of the symmetric group

$$\phi : \mathfrak{S}_n \longrightarrow GL_{n-1}(\mathbb{R})$$

and define the Waldspurger matrix, $\mathbf{W}(g)$, of a permutation g to be the matrix of $\phi(1) - \phi(g)$ expressed in root coordinates. We have a slick trick for computing $\mathbf{W}(g)$, which exposes beautiful combinatorial structure.

Column vectors of Waldspurger matrices are in bijection with unimodal-Motzkin paths and relate to abelian ideals of the Lie Algebra \mathfrak{sl} . $\mathbf{W}(g)$ has distinct non-zero columns iff g is a SIF permutation as defined by Callan.

The cones over columns of Waldspurger matrices give a surprising decomposition of the closed root cone. What's more, Waldspurger matrices give a tiling of \mathbb{R}^n . Define the Meinrenken simplex, $\mathbf{M}(g)$, of a permutation g to be the convex hull of the zero vector and the columns of $\mathbf{W}(g)$. Meinrenken showed $\mathbf{MT}(g) := \bigsqcup_{g \in \mathfrak{S}_n} \mathbf{M}(g)$ forms a (non-convex!) tile for Euclidean space under the action of the root lattice. The intersection of $\mathbf{M}(g)$ and the boundary of \mathbf{MT} is the convex hull of the columns of $\mathbf{W}(g)$. This restriction to the boundary has curious enumerative properties. (Received August 24, 2016)