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Michael Dairyko, Michael Ferrara, Bernard Lidický, Ryan Martin, Tyrrell McAllister, Florian Pfender and Andrew Uzzell* (andrew.uzzell@unl.edu). *Degree conditions for small contagious sets in bootstrap percolation.*

Bootstrap percolation is a cellular automaton introduced in 1979 by Chalupa, Leath, and Reich. Fix $r \geq 2$. In r -neighbor bootstrap percolation on a graph G , all vertices are either “infected” or “uninfected”. In this process, an initially infected set $A \subseteq V(G)$ grows by iteratively infecting all uninfected vertices with at least r infected neighbors. If all vertices eventually become infected, we say that the initial set A is r -contagious.

Let $m(G, r)$ denote the minimum size of an r -contagious set in G . Clearly, $m(G, r) \geq \min\{|V(G)|, r\}$. What conditions on G imply that $m(G, r) = r$? Let $\sigma_2(G) = \min\{d(x) + d(y) : xy \notin E(G)\}$. Freund, Poloczek, and Reichman recently showed that if $\sigma(G) \geq n$, then $m(G, 2) = 2$. We show that in fact, $\sigma_2(G) \geq n - 2$ is almost enough to imply $m(G, 2) = 2$: if $\sigma_2(G) \geq n - 2$ and $m(G, 2) > 2$, then either G is in one of four infinite families or G is one of nine exceptional graphs. We also show that if G is a graph with degree sequence $d_1 \leq \dots \leq d_n$ such that for all $1 \leq i < n/2$, either $d_i \geq i + 1$ or $d_{n-i} \geq n - i - 1$, then either $m(G, 2) = 2$, $G \cong C_5$, or G is in one of two infinite families. (Received August 29, 2016)