

1123-49-280

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Critical Configurations of Hard Disks on S^2 . Preliminary report.

We study the *injectivity radius* or *thickness* function r on the configuration space $C(N, S^2)$ of N distinct points on the unit 2-sphere. Criticality for maximizing r is equivalent to the existence of a *balanced contact graph* of geodesic arcs whose vertices are (a subset of) the points in the configuration (often, but not always, the remaining points are “rattlers”). We also develop a Morse Lemma for the second order behavior of r near such a critical configuration: $r = q + p + o(2)$, where q is a quadratic function on the tangent space of $C(N, S^2)$, and where p is piecewise linear and concave. In general, such critical configurations comprise a semi-algebraic subvariety of $C(N, S^2)$ and the corresponding critical values are a finite subset of the interval $[\pi/N, \pi]$. For small values of N , we describe all the critical configurations and the corresponding Morse Complex; we also aim to understand special values of N , like $N = 12$, where some surprises occur. (This is part of a joint project with W. Kusner, J. Lagarias and S. Shlosman.) (Received August 29, 2016)