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**Blake Dunshee\*** ([blake.dunshee@belmont.edu](mailto:blake.dunshee@belmont.edu)), **Mark Ellingham** and **Joanna A Ellis-Monaghan**. *Directed embeddings with partially predetermined facial walks.*

A directed embedding is a digraph embedded in a surface in such a way that all the faces of the embedding are bounded by directed walks. This is equivalent to an embedding of a digraph where half-arcs alternate between in and out around a vertex. First, we characterize when an embedded graph can be given an orientation such that the resulting embedding of a digraph is a directed embedding. We then characterize when a digraph and a collection of its closed directed walks can be extended to a directed embedding such that the collection of closed directed walks is a subcollection of the facial walks. Furthermore, we characterize when such a directed embedding can be chosen to be orientable. Širáň and Škoviera gave analogous results in the case of undirected graphs by characterizing when a graph and a collection of its closed walks can be extended to an embedding such that the collection of closed walks is a subcollection of the facial walks. The necessary and sufficient conditions are stronger in the case of orientable directed embeddings than those for orientable embeddings of undirected graphs. (Received September 11, 2020)