

1163-11-1439

**Harald Andres Helfgott** and **Lola Thompson\*** (l.thompson@uu.nl). *Summing  $\mu(n)$ : an even faster elementary algorithm.*

We present a new-and-improved elementary algorithm for computing  $M(x) = \sum_{n \leq x} \mu(n)$ , where  $\mu(n)$  is the Möbius function (an older version of this paper, with a somewhat different approach and larger running time, was presented at the 2020 Joint Mathematics Meetings). Our algorithm takes

$$\text{time } O\left(x^{\frac{3}{5}} \log \log x\right) \text{ and space } O\left(x^{\frac{3}{10}} \log x\right),$$

which improves on existing combinatorial algorithms. While there is an analytic algorithm due to Lagarias-Odlyzko with computations based on integrals of  $\zeta(s)$  that only takes time  $O(x^{1/2+\epsilon})$ , our algorithm has the advantage of being easier to implement. The new approach roughly amounts to analyzing the difference between a model that we obtain via Diophantine approximation and reality, and showing that it has a simple description in terms of congruence classes and segments. This simple description allows us to compute the difference quickly by means of a table lookup. (Received September 15, 2020)