1163-11-29 **Michael David Fried*** (michaeldavidfried@gmail.com). *Every finite group challenges extending Falting's Theorem.*

Consider finite group G; ℓ a prime dividing $|G|$ ($|G$ has no $\mathbf{Z}/\ell$ quotient); and $\mathbf{C} = \{\mathrm{C}_1 \ldots \mathrm{C}_r\}$ any $r \geq 4$ conjugacy classes of order prime to ℓ elements. Ex: $G = A_5$, $\mathbf{C}$ is 4 repetes of the 3-cycle conjugacy class, and $\ell = 2$.

For $(G, \mathbf{C}, \ell)$, $M' \in I$, $|I| < \infty$ gives a $\mathbf{Z}_\ell[G]$ lattice $L_{M'}$ as kernel of an ℓ-Frattini cover $\tilde{G}_{M'} \to G \implies$ a moduli space series

$$\cdots \to \mathcal{H}(G, \mathbf{C}, \ell, L)_k \to \cdots \to \mathcal{H}(\ldots)_1 \to \mathcal{H}(G, \mathbf{C}, \ell, L)_0 \to J_r.$$

Terms are quadi-projective varieties. When $r = 4$ all are upper half plane quotients; J_4 is the classical j-line, minus ∞.

Only for G "close to" dihedral ($r = 4$) are these modular curves.

Main Conjecture: Let K be any number field. For k large, projective normalization of $\mathcal{H}(G, \mathbf{C}, \ell, L)_k$ has general type, and $\mathcal{H}(G, \mathbf{C}, \ell, L)_k$ has no K points.

For $r = 4$, there are two proofs (myself/Cadoret-Tamagawa). We compare these results and how even this presents an unproven challenge to extending Falting's Theorem. (Received July 08, 2020)