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**Scott T. Chapman\*** ([scott.chapman@shsu.edu](mailto:scott.chapman@shsu.edu)), Department of Mathematics and Statistics, Box 2206, Huntsville, TX 77341, and **Christopher O’Neill** and **Vadim Ponomarenko**. *On Length Densities*.

For a commutative cancellative monoid  $M$ , we introduce the notion of the *length density* of both a nonunit  $x \in M$ , denoted  $\text{LD}(x)$ , and the entire monoid  $M$ , denoted  $\text{LD}(M)$ . This invariant is related to three widely studied invariants in the theory of non-unit factorizations,  $L(x)$ ,  $\ell(x)$ , and  $\rho(x)$ . We consider some general properties of  $\text{LD}(x)$  and  $\text{LD}(M)$  and give a wide variety of examples using numerical semigroups, Puiseux monoids, and Krull monoids. While we give an example of a monoid  $M$  with irrational length density, we show that if  $M$  is finitely generated, then  $\text{LD}(M)$  is rational and there is a nonunit element  $x \in M$  with  $\text{LD}(M) = \text{LD}(x)$  (such a monoid is said to have accepted length density). While it is well-known that the much studied asymptotic versions of  $L(x)$ ,  $\ell(x)$  and  $\rho(x)$  (denoted  $\bar{L}(x)$ ,  $\bar{\ell}(x)$ , and  $\bar{\rho}(x)$ ) always exist, we show the somewhat surprising result that  $\overline{\text{LD}}(x) = \lim_{n \rightarrow \infty} \text{LD}(x^n)$  may not exist. We also give some finiteness conditions on  $M$  that force the existence of  $\overline{\text{LD}}(x)$ . (Received August 17, 2020)