1163-52-358 **Jesus A. De Loera*** (deloera@math.ucdavis.edu), Dept. of Mathematics, University of California, One Shields Avenue, Davis, CA 95616. *Combinatorial Challenges arising from the Simplex Method.*

Dantzig's Simplex method is a work horse of modern optimization. But despite its success we do not understand its complexity. To bound the number of iterations of the Simplex method we take a geometric point of view and investigate the lengths of monotone paths inside the oriented graphs of polyhedra (oriented by the objective function). We consider the shortest and the longest monotone paths possible and estimate the (monotone) diameter and the height of some famous combinatorial polyhedra (such as TSP, fractional matching polytopes, and others). Surprisingly, as we look at all monotone paths put together we see a rich topological CW-space structure which can be used to count how many are there or to generate them randomly. Our main enumerative results include bounds on the number of monotone paths, and on the the diameter of the CW-complex of monotone paths (how far are two monotone paths from each other?). The picture is fairly complete in dimension three, but an open problem for high dimensional polytopes.

Joint work with Moise Blanchard (MIT) and Quentin Louveaux (U. Liege) and Christos Athanasiadis (U. Athens) and Zhenyang Zhang (UC Davis). (Received September 03, 2020)