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I propose a way that (concrete) group theory can illuminate some basic ideas of school math, through studying additive and multiplicative groups of real numbers. The key is Division with Remainder: (DwR) For  $a$  and  $b > 0$  in  $\mathbb{R}$ ,  $a = qb + r$  with  $q$  in  $\mathbb{Z}$  (integers) and  $0 \leq r < b$ . Let  $A$  be a subset of  $\mathbb{R}$  and  $a$  in  $A$ . Call  $a$  isolated in  $A$  if, for some  $r > 0$ ,  $(a-r, a+r)$  contains no element  $\neq a$  from  $A$ . Call  $A$  discrete if all of its elements are isolated, and uniformly discrete if some  $r > 0$  works for all  $a$  in  $A$ . Call  $A$  an additive group if  $A$  is closed under  $+$  and  $-$ . Let  $A$  be a real additive group. The following follow easily from (DwR). (I) If  $0$  is isolated in  $A$  then  $A$  is uniformly discrete. (II)  $A$  is either discrete or dense in  $\mathbb{R}$ . (III) If  $A$  is discrete then  $A = \mathbb{Z}a$  for a unique  $a \geq 0$ . (IV)  $\mathbb{Z}a + \mathbb{Z}b$  is discrete iff  $a$  and  $b$  are commensurable. In this case,  $d = \gcd(a, b) \geq 0$  and  $m = \text{lcm}(a, b) \geq 0$  are defined by,  $\mathbb{Z}a + \mathbb{Z}b = \mathbb{Z}d$  and  $\mathbb{Z}a \cap \mathbb{Z}b = \mathbb{Z}m$ . Using the continuous group isomorphism  $\exp: \mathbb{R} \rightarrow (0, \infty)$ , one can similarly describe discrete multiplicative subgroups of  $\mathbb{R}$ . Studying the additive and multiplicative structures of  $\mathbb{C}$  and of  $\mathbb{Z}/\mathbb{Z}m$  is more complex.. (Received September 10, 2020)