In Ramsey Theory one studies the notion of partition regularity (PR): A family \( G \) is PR on \( X \) if for every finite coloring \( X = C_1 \cup \ldots \cup C_r \) there exists a monochromatic \( A \in G \), i.e., \( A \subseteq C_i \) for some \( i \). In this setting, ultrafilters play an instrumental role: A family \( G \) is PR on \( X \) iff there exists an ultrafilter \( U \) on \( X \) such that every element of \( U \) includes some \( A \in G \). By using the nonstandard extension \( ^*X \), ultrafilters on \( X \) can be represented as points \( \xi \in ^*X \). I will present a nonstandard technique grounded on that observation, which has been recently used to prove new results about the PR of Diophantine equations. (An equation \( P(x_1, \ldots, x_n) = 0 \) is PR if the set of its solutions \( \{ (a_1, \ldots, a_n) \mid P(a_1, \ldots, a_n) = 0 \} \) is PR.) As examples, I will show that \( X^2 + Y^2 = Z \) and \( X + Y = Z^2 \) are not PR.

In the second part of the talk, I will briefly discuss the (discrete) topological dynamics as given by the hypernatural numbers \( ^*\mathbb{N} \) endowed with the shift operator \( S : \xi \mapsto \xi + 1 \), and present an alternative nonstandard proof of van der Waerden’s Theorem: In any finite coloring of the natural numbers there exist monochromatic arithmetic progressions of arbitrary length. (Received January 14, 2019)