The Catalan numbers $1, 2, 5, 14, \ldots$ is one of the most well-known sequences in combinatorics. It enumerates over 100 families of combinatorial objects. Some of these families include the set of Dyck paths in an $n \times (n+1)$ rectangle, the set of $(+n, +n+1)$-invariant subsets of $\mathbb{N}$ containing 0, simultaneous $(n, n+1)$-cores, the $(n+1)$-restricted affine permutations in $\hat{S}_n/S_n$, the number of cells in a certain affine Springer fibre, a basis of the representation $eL_{(n+1)/n}$ of the spherical Cherednik algebra $eH_n e$. The above families and the bijections between them all generalize from $(n, n+1)$ to $(n, m)$ when $\gcd(n, m) = 1$. However when $\gcd(dn, dm) = d > 1$, many of these break: in particular some of the sets stay finite while others become infinite.

I will discuss an equivalence relation on the infinite set of $(+dn, +dm)$-invariant subsets of $\mathbb{N}$, such that its equivalence classes are again in bijection with finite set of Dyck paths in a $dn \times dm$ rectangle. Our hope is that this construction will lead to a geometric or representation theoretic interpretation of the dinv statistic from the $d = 1$ case.

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