Nonlinear Averaging, Nonlinear PDEs and Statistical Functional Equations.

Considering the mean-value property of harmonic functions, it is natural to ask if solutions of other PDEs can be characterized statistically. Asymptotic statistical formulas, involving the median, do characterize $p$-harmonic functions when $1 < p < \infty$, but an example shows that the direct analog of the mean-value property arising from these formulas does not hold in general. Nonlocal functional equations related to these asymptotic formulas will be considered, the solutions of which are known as $p$-harmonious functions. Using largely elementary methods, it can be shown that continuous $p$-harmonious functions with prescribed Dirichlet data are unique and exist when a sub/supersolution pair can be found. The solutions of these problems approximate $p$-harmonic functions, thus establishing a further link between nonlinear averages and the solutions of nonlinear PDE. My joint work with Matthew Rudd will be presented, and connections with some of the earlier results of others, particularly Manfredi, Parviainen and Rossi, will be discussed. The $p = 1$ case and connections to median values and functions of least gradient will also be considered. The talk will include a survey of some recent work in this direction and a discussion of topics of ongoing investigation. (Received January 25, 2019)