For the classical Sobolev inequality on Euclidean space, equality is achieved for precisely one (explicit) function and its dilations, translations, and constant multiples. The question of stability asks whether a function that almost attains equality in the Sobolev inequality must be close to some scaling of this extremal function. We establish a strong-form quantitative stability estimate for all $p \in (1, n)$ in which we measure the distance from a function to the family of extremal functions in terms of the $L^p$ norm between their gradients. (Received January 25, 2019)