Let $G$ be $SL_2(R)$, the special linear group of degree 2 over the real numbers and $\sigma$ denote a bounded Euler 2-cocycle on $G$ with values in the real numbers. The resulting cohomology class corresponds to the central extension given by universal covering $\tilde{G}$ of $G$, thus the pullback of $\sigma$ is a coboundary on $\tilde{G}$ of a 1-cochain $\tau$ of $\tilde{G}$. The pair $(\sigma, \tau)$ gives rise to a nontrivial cohomology class in the relative cohomology group of $(G, \tilde{G})$. Let $M$ denote a binary tube, a surface consisting of trivalent tubes or a two-holed disk. Given a flat $G$-bundle on $M$, the relative cohomology class makes a pairing with a relative homology group of $(M, \partial M)$. One also has an evaluation of the Euler cocycle with summable group cycles of $G$ since the cocycle is bounded. I shall talk about an index theorem on binary tube involved with the relative pairing and evaluation of the Euler cocycle, in which Poincare’s rotation number of circle diffeomorphism appears. (Received January 27, 2019)