Toric varieties form an important class of algebraic varieties that are among the simplest objects in algebraic geometry. In classical theory normal toric varieties are given by rational fans in $\mathbb{R}^n$. Motivated by applications, we construct a theory of irrational toric varieties associated to arbitrary fans in $\mathbb{R}^n$. These are $\mathbb{R}^n$-equivariant cell complexes dual to the fan and generalize the nonnegative part of a classical toric variety. Among the pleasing parallels with the classical theory is that the space of degenerations of a projective irrational toric variety is homeomorphic to the secondary polytope of a point configuration. (Received February 19, 2018)