In non-commutative geometry cosemisimple Hopf algebras are treated analogous to the algebras $O(G)$ of regular functions on linearly reductive affine algebraic groups $G$. In this context, a homogeneous space of the “quantum group” attached to a Hopf algebra $H$ would be a coideal subalgebra of $A$. Similarly, a quantum subgroup would be a quotient Hopf algebra $H \to C$.

Classically, the quotient stack of an algebraic group $G$ by a closed subgroup $N$ is an affine scheme precisely when $O(G)$ is faithfully flat over the coideal subalgebra $O(G/N)$ consisting of functions constant along the cosets of $N$. For this reason, faithful flatness is the technical condition that encapsulates the vague notion that homogeneous spaces are “well behaved”.

For Hopf algebras $H$ equipped with enough structure to render them analogous to function algebras on compact quantum groups it turns out that inclusions of right coideal $*$-subalgebras are automatically faithfully flat. In view of the previous paragraph, this confirms the intuition that quotients of semisimple affine algebraic groups by semisimple closed subgroups are affine schemes. (Received February 12, 2018)