For any integer $n \geq 2$ we construct a one-round two-player game $G_n$, with communication that scales poly-logarithmically with $n$, having the following properties. First, there exists an entangled strategy that wins with probability 1 and in which the players perform generalized Pauli measurements on their respective share of an $n$-qudit maximally entangled state, with local dimension $q = \text{poly log}(n)$. Second, any strategy that succeeds with probability at least $1 - \varepsilon$ must be within distance $O((\log n)^{c\varepsilon^{1/d}})$, for universal constants $c, d \geq 1$, of the perfect strategy, up to local isometries. This is an exponential improvement on the size of any previously known game certifying $\Omega(n)$ qudits of entanglement with comparable robustness guarantees. The construction of the game $G_n$ is based on the classical test for low-degree polynomials of Raz and Safra, which we extend to the quantum regime. We further obtain several consequences for complexity theory, most notably that it is QMA-hard, under randomized reductions, to approximate up to a constant factor the maximum acceptance probability of a multiround, multiplayer entangled game with poly log($n$) bits of classical communication. (Received February 14, 2018)