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Pittsburgh, PA 15213. *Proofs as objects*.

The rigor of mathematics lies in its systematic organization; it makes for conclusive proofs of assertions on the basis of assumed principles. Proofs are constructed through thinking, but can also be taken as objects of mathematical thought. That was the key insight underlying Hilbert's call in 1917 for a theory of the specifically mathematical proof.

Hilbert's call was rooted in revolutionary developments in 19-th century mathematics and logic: the introduction of abstract concepts in mathematics and the formulation of symbolic calculi in logic. The resulting formal axiomatic frames allowed the full formalization of mathematical practice and served later as tools for Hilbert's finitist consistency program.

A theory of the specifically mathematical proof must examine the processes underlying proof construction with the help of natural formalization and automated search. The combination of interactive verification and mechanical theorem proving is a promising avenue for exploring mathematical thought.

As case studies, I will discuss the natural formalization of the Cantor-Bernstein Theorem and the automated search for Gödel's Incompleteness Theorems. That experience is the basis for some reflections on the uniqueness and identity of proofs. (Received July 15, 2019)