Spanning trees, $G$-parking functions, and the Tutte polynomial are familiar characters in combinatorics. In an effort to generalize bijective correspondences between them, we define an object called a tree growing sequence. The definition is inspired by ”splitting” operations arising from the recursive formula for computing the Tutte polynomial. One can apply a tree growing sequence to a $G$-parking function in a way that simultaneously assigns to it both a spanning tree and a monomial. We construct maps from a few known algorithms to the collection of tree growing sequences, and discuss commutative diagrams involving these maps. A relation to splitting operations for zonotopal tilings will be briefly described. (Received July 29, 2020)