For a fixed integer $k \geq 2$, let $q$ be a prime power such that $q \equiv 1 \pmod{k}$ if $q$ is even, or $q \equiv 1 \pmod{2k}$ if $q$ is odd. The generalized Paley graph $G_k(q)$ is the graph with vertex set $\mathbb{F}_q$ and edge set \{\text{\textit{ab}} : a - b \text{ is a } k\text{-th power residue modulo } q\}. We give a formula in terms of finite field hypergeometric functions for the number of complete subgraphs of order four contained in $G_k(q)$, and a formula in terms of Jacobi sums for the number of complete subgraphs of order three contained in $G_k(q)$, for all $k$. This generalizes work of Evans, Pullham, and Sheehan on the Paley graph with $k = 2$. We apply these formulas to obtain lower bounds for the multicolor diagonal Ramsey numbers $R_k(4)$ and $R_k(3)$. We also examine connections to Fourier coefficients of modular forms, when $q$ is prime. (Received August 05, 2020)