Differential operators on singular rings have found many applications, but are generally quite hard to describe explicitly. A natural question is "how transitive" the action of the differential operators on the ring is, and specifically if there's a nontrivial submodule. If not, the ring is called $D$-simple. In many cases, $D$-simplicity corresponds to "milder" singularities: for example, smooth rings are $D$-simple, as are their invariant subrings under finite group actions. In this talk, we'll review results connecting $D$-simplicity and mildness of singularities in characteristic $p$, which lead to a natural expectation for a connection in characteristic 0. However, we'll show that this natural expectation is unfortunately not true. To prove this, we'll discuss the connection of $D$-simplicity of graded rings with mild singularities to the global geometry of Fano varieties, and point to some outstanding problems on positivity of their tangent bundles. (Received August 10, 2020)