In algebraic geometry, it is a common phenomenon that moduli space of geometric objects tend to become more and more complicated as invariants parametrizing them go to infinity. In this talk, I would like to discuss this phenomenon for the space of singular curves on K3 surfaces.

Let $(X, L)$ be a very general polarized K3 surface of genus $g$ so that $L^2 = 2g - 2$. A curve $C$ in the linear system $|L|$ is a rational curve if and only if it has $g$ nodes. The Yau-Zarslow formula implies that the number of rational curves goes to infinity as $g$ goes to infinity.

We would like to explore the next case: curves in the linear system $|L|$ with $(g - 1)$-nodes. They are singular elliptic curves and let $M$ be the space of such curves. It can be shown that $M$ is one dimensional, hence we can talk its geometric genus. With Nathan Chen and Francois Greer, we show that the geometric genus of $M$ goes to infinity as $g$ goes to infinity. (Received July 31, 2020)