Motivated by open questions in the long-time dynamics and fluid turbulence, we investigate the existence of nonstationary solutions to the Navier-Stokes equations (NSE) with constant energy profiles, with constant enstrophy profiles, or both. In this talk we make progress in this area by constructing a nonstationary solution to the incompressible NSE (on the 3D torus) whose energy remains constant. Similarly, we construct a nonstationary solution whose enstrophy remains constant. These constructions necessarily exist outside of the attractor and are supported on an infinite number of Fourier modes. On the 2D torus we show that nonstationary constant energy (resp. enstrophy) solutions (if they exist) must necessarily be supported on an infinite number of modes when the force is an eigenvector of the Stokes operator. (Received August 31, 2020)