In this talk, I will introduce a homotopy training algorithm (HTA) to solve optimization problems arising from fully connected neural networks with complicated structures. The HTA dynamically builds the neural network starting from a simplified version and ending with the fully connected network via adding layers and nodes adaptively. Therefore, the corresponding optimization problem is easy to solve at the beginning and connects to the original model via a continuous path guided by the HTA, which provides a high probability of obtaining a global minimum. Then I will apply the HTA to compute the multiple solutions of nonlinear PDEs with neural network discretization. (Received January 23, 2020)