## Heba Ayeda, David Beecher, Xiaoxiao Cui* (xiaoxiaocui@cpp.edu), Alan Krinik,

 Jeremy Lin, Thuy Vu Dieu Lu, David Perez, Randall J. Swift and Weizhong Wong.Gambler's ruin probabilities for finite birth-death chains with alternating probabilities and the ballot box problem for couples.
PROBLEM 1. Consider a finite birth-death chain on states: $0,1,2, \ldots, H-1, H$ having absorbing states at 0 and $H$, alternating ascending probabilities, alternating descending probabilities and alternating return probabilities at states $1,2,3, \ldots, H-1$. Then for each state $i, 0<i<H$ and for each $n \in \mathbb{N}$, we determine the exact $n$-step ruin (transition) probability $P^{(n)}(i, 0)$ in terms of explicit eigenvalue expressions. This result follows from duality theory and eigenvalue results for tridiagonal matrices having alternating entries (Kouachi, S. 2008). PROBLEM 2. The traditional two candidate Ballot Box Problem is extended to include couple voters. This model assumes the relevant transition matrices to be symmetric, penta-diagonal, Toeplitz and sub-stochastic. The closed form solution of this problem is obtained in terms of explicit eigenvalue expressions for transition matrices of size 8 X 8 or less. (Received September 21, 2021)

