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KURT O. FRIEDRICHS

LECTURE NOTES

Mathematical Methods of Electromagnetic Theory

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Contents

Foreword vii

Introduction ix

Chapter 1. Preliminaries 1
1.1. Notions of Vector Analysis 1
1.2. Notions of Mechanics 4

Chapter 2. Electrostatics 7
2.1. The Electric Field 7
2.2. Fields Generated by Lines, Surfaces, and Space Regions 12
2.3. Differential Laws of Electrostatics 20
2.4. Dielectric Media 24
2.5. Conductors 28
2.6. Boundary Problems 32
2.7. Potential Energy 36
2.8. Two-Dimensional Problems of Electrostatics 40
2.9. The Electric Field Outside a Cylinder 44

Chapter 3. Currents and Ohm's Law 49
3.1. Currents 49
3.2. Resistive Conductors. Ohm's Law. 52

Chapter 4. Magnetostatics 59

Chapter 5. Electromagnetic Fields Changing in Time 71
5.1. The Basic Laws of Electromagnetism 71
5.2. Magnetic Inductions 75
5.3. Pre-Maxwellian Magnetic Induction 77
5.4. Inductance 79
5.5. Special Induction Loops 81

Chapter 6. Transmission Lines. Method of the Laplace Transformation 87
6.1. Transmission Lines 87
6.2. Undistorted Transmission 90
6.3. The Method of Laplace Transformation 92
6.4. Special Transmission Problems 95
6.5. Extensions of the Method of Laplace Transformations 101
6.6. Heaviside Calculus 103

Chapter 7. Electromagnetodynamics of Moving Bodies and the Principle of Relativity 107
7.1. Extended Magnetic Induction 107
7.2. Maxwell's Equations with Reference to a Moving Coordinate System 109
7.3. Maxwell's Equations in Covariant Form 111
7.4. The Principle of Relativity 116

Chapter 8. Electromagnetic Wave Propagation 119
8.1. Propagation with Finite Speed 119
8.2. Initial Value Problems 121
8.3. Huygens' Principle 125
8.4. Electromagnetic Potentials 126
8.5. The Radiation Problem 128

Chapter 9. The Scattering Problem 135
9.1. Bundles and Packets of Plane Waves 135
9.2. Scattering Induced by a Conductor 136
9.3. Scattering of Electromagnetic Waves 140

References 145

Foreword

Kurt Friedrichs was one of the foremost analysts and mathematical physicists of the twentieth century. His remarkable book on electromagnetic theory is not a historical curiosity, although it reflects much of his unique style and carries a scent of the nineteenth century. It is, in fact, a clear and intuitive, but precise, explanation of the phenomena governed by Maxwell's equations. Unlike the treatment in many texts, there is little reliance on "physical shortcuts" in addressing questions relating to the uniqueness of solutions or to the interaction of electromagnetic waves with matter. Particularly noteworthy are his use of variational principles in solving the problems of electro- and magnetostatics, his derivation of energy methods, and his ability to connect basic theory to predictions that can be tested experimentally. Friedrichs' discussion of time-varying fields is superb, with an emphasis on the central role of charge conservation, the finite speed of wave propagation, and scattering theory. It also includes a beautiful (but short) introduction to special relativity and its intimate connections to Maxwell's equations.

While some mathematical sophistication is required, the book is accessible to students who have taken both advanced calculus and linear algebra and have had some exposure to classical mechanics and electromagnetics at the undergraduate level. There are surprisingly few texts that, like this one, are both rigorous and accessible. It is a modern classic.

Charles Epstein, *University of Pennsylvania*
Leslie Greengard, *Courant Institute*

Introduction

The present course will deal with electromagnetic theory from a mathematical point of view. That does not mean that this theory is used just as a playground for mathematical methods or simply as a reservoir of problems to which mathematical theorems can be applied. Electromagnetic theory will be taken for what it is, a theory of physics. Still the attitude in dealing with this theory may be different from the one naturally taken by a physicist or an electro-engineer.

Take the question: what is electricity? In one book this is said: "If a stick of sealing-wax is rubbed with cat's fur, both bodies are put into a peculiar condition in which light bodies in their neighborhood are set in motion." In another book it is taken for granted that there is a socket in the baseboard in your room and electricity is described by the peculiar happenings that can take place at the end of a pair of wires plugged into the socket. If, in particular, one puts an electric razor at the end of the pair of wires and observes that it runs, one will be led to the definition: what makes the razor run, that is electricity.

We shall not adopt such a "technological" definition of electricity, nor the "romantic" sealing-wax and cat's fur definition. We shall characterize in rather general terms electricity by the specific manner in which electric forces act on objects and depend on the agent that produces them. We shall show that in this manner of action and dependence electric forces differ in a purely formal way from gravitational and elastic forces. Thus our definition of electricity might be called an axiomatic one, although this term has some unwanted connotations. In explaining the phenomena of electricity and magnetism, physicists frequently follow the way in which an understanding of these phenomena has developed historically, or they present the simplest realistic experiments that exhibit these phenomena most pointedly. We shall assume the present-day understanding of the theory (or more precisely, the understanding reached after the advent of the special theory of relativity in 1905), and we shall exemplify the basic laws, not by the simplest possible realistic experiments, but by simple fictitious experiments that represent these laws most directly without technological encumbrances.

This approach is not meant to be "the" right approach—far from it. It is meant to lead to one out of several aspects of the theory of electricity and magnetism, all of which together contribute to an understanding of these phenomena.

The title of the course involves the notion of methods. To be sure, there are mathematical methods for solving mathematical problems that express physical questions. A "solution" of such a problem may lead to a numerical evaluation, but

frequently it will help us to understand the phenomena and lead to overall expectations. We shall interpret the term "method" in still a wider sense. Specifically, this term is also meant to cover mathematical methods of proper formulation of physical laws; thus it includes the vector and tensor analysis in Euclidean and Minkowskian space; but more generally it is to cover mathematical results that show what kind of differential equations are suitable as expressions of physical laws.

The term "physical laws" here refers not only to the basic ones, but also to laws derived by various simplifying assumptions. We shall frequently discuss the question how such simplifications can be justified, but we shall rarely do this rigorously. In fact, we shall not at all indulge in the sport of being rigorous in the strict sense in which this notion is understood in mathematics.

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Mathematical Methods of Electromagnetic Theory

KURT O. FRIEDRICHS

This text provides a mathematically precise but intuitive introduction to classical electromagnetic theory and wave propagation, with a brief introduction to special relativity. While written in a distinctive, modern style, Friedrichs manages to convey the physical intuition and 19th century basis of the equations, with an emphasis on conservation laws. Particularly striking features of the book include: (a) a mathematically rigorous derivation of the interaction of electromagnetic waves with matter, (b) a straightforward explanation of how to use variational principles to solve problems in electro- and magnetostatics, and (c) a thorough discussion of the central importance of the conservation of charge. It is suitable for advanced undergraduate students in mathematics and physics with a background in advanced calculus and linear algebra, as well as mechanics and electromagnetics at an undergraduate level. Apart from minor corrections to the text, the notation was updated in this edition to follow the conventions of modern vector calculus.

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