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Tactile Learning Activities in Mathematics

A Recipe Book for the Undergraduate Classroom

Julie Barnes
Jessica M. Libertini

MAA PRESS
An Imprint of the AMERICAN MATHEMATICAL SOCIETY

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We greatly appreciate the efforts of the individuals and groups who helped proofread or test some of the activities; these people include Karen Bliss, Meagan Herald, Kim Johnson, Ben Kearns, Laura Lembeck, Lisa Lyford, John Wagaman, and the Women's Writing Group at Virginia Military Institute. We would also like to thank the Western Carolina University Math Club for agreeing to pose for photographs.

This book includes activities from talented faculty members that work at a wide range of institutions, represent all career stages, and are from a variety of geographical regions across the United States. We would like to thank all of our contributing authors for sharing their ideas in this book, and for working so cooperatively with us throughout the editing process.

We would like to thank everyone involved with the Classroom Resource Materials series of the MAA's publishing division for their encouragement and support throughout the process, including editors Jerry Bryce and Susan Staples, as well as the thoughtful team of anonymous reviewers who went through this manuscript and provided us with clear and helpful guidance that shaped the revision process.

Finally, we would like to share our deepest gratitude for our dear friends and family for their support, encouragement, and understanding as we worked on this project.

Preface

Tell me, and I forget. Teach me, and I may remember. Involve me, and I learn.

—Ancient Proverb

Suppose you are scheduled to visit a college math classroom and find feather boas, hula hoops, cards, balls, jacks, or a large stack of cookies. Perhaps you think the class is somewhere else, and this classroom is set up for young children. Thinking back on your own preschool days, you know that using tactile activities for learning is commonly accepted and well documented in early education; the learning theories of Froebel, Montessori, Piaget, and Erikson all indicate the importance of hands-on learning. While you may have heard of experiential learning at the college level, that term typically refers to internships or lab classes, and you are fairly certain it doesn't apply to mathematics. So you check the sign on the door, and sure enough, this is the correct math class, taught by any one of the contributors to this book, including the two of us.

The value of active learning, which includes tactile learning, is well documented in the literature. The benefits of these active learning approaches include increased accessibility, deeper understanding, and a way to level the playing field for students with diverse backgrounds and levels of mathematical preparation [3, 5, 7, 8]. In addition to the strong evidence in the literature, both of us have had the joy of observing our students dissect, discuss, distill, and discover mathematics through tactile learning. We encourage you, the curious instructor, to look through the literature to get a clearer understanding of how activities can enhance learning in your classes.

The two of us have been using a variety of tactile activities in our classes for years, and we are constantly seeking and developing new ideas. We co-organized an MAA Special Session, "Touch it, feel it, learn it: Tactile learning activities in the undergraduate mathematics classroom," for the 2012 JMM in Boston. With 35 presentations drawing an audience of 100-150 people, we decided to offer a second MAA Special Session at the 2013 JMM in San Diego, where we had an additional 26 well-attended presentations. Given the diversity of the presentations at these sessions, it is clear that there a a large number of mathematics faculty members using a wide variety of tactile activities in their classes, and an even larger number of people showing interest in the subject.

In response to the high level of interest from the mathematical community, we guest-edited a special issue of *PRIMUS* [2] dedicated to hands-on activities in which the authors, many of whom have also contributed to this book, offered specific pedagogical support for the activities presented therein. We have also assembled this book, offering a diverse collection of activities targeting mathematical topics ranging in level from precalculus to knot theory. We developed this book for you, the curious instructor—both the seasoned experimentalists and those trying hands-on activities for the first time, and we hope that you find it beneficial as you invite your students to explore mathematics tactilely in your classrooms.

We are sure that all of the people who spoke in our sessions, attended the talks, authored articles in our *PRIMUS* issue, or contributed to this book have their own tales about what initially sparked their interest in teaching this way. We include our stories below.

Jessica's story: My calculus class at West Point was just getting ready to transition from single variable integration to integrating functions of two variables, and I knew that my students would struggle with the concept of "the volume beneath a surface". I spent my entire afternoon writing, scrapping, re-writing, and re-scrapping lesson plans, Mathematica notebooks, and chalkboard sketches to make the concept accessible, but nothing seemed concrete enough. The night before the lesson, I went to bed very worried. As I sleeplessly tossed and turned, I caught sight of my arm moving under the sheet, creating a beautiful surface. I sat straight up, realizing that my students could stand in formation, and by tossing a sheet over them, my class could create their own surface, using their own heights to approximate the integral. I tried this activity in class the next day, and based on the positive feedback from my students and the prodding of a colleague, I wrote an article on this activity that appeared in *PRIMUS* [6].

The year after developing the sheet activity for my calculus class, I was asked to develop a new course called Mathematics for Space Applications; there was no similar course in the country at the time. Drawing students from systems engineering, mathematics, physics, and a handful of other majors, I had difficulty designing lectures that addressed the broad backgrounds of my students. Shortly into the course, I tossed out the majority of my lectures and went to a full peer-teaching and hands-on learning approach. We represented orbits using hula hoops and globes; we made model solar systems with sidewalk chalk and string; we spun ourselves silly in office chairs as we traced out highly elliptical orbits with our feet; and most importantly, we had fun while we learned. I wrote an article on the format of this course, including the hands-on activities, for *Mathematica Militaris* [4]. Through teaching this course, I developed a new appreciation for hands-on activities as a way of leveling the playing field for a diverse group of students, and I continue to leverage this benefit in my teaching at the Virginia Military Institute.

Julie's story: After covering the epsilon-delta definition of continuity in real analysis, I noticed many pairs of glazed eyes staring back at me. The definition had been too abstract for the students, so I tried to explain the definition again. I drew a diagram, but it was too static. I used technology to zoom in on a graph, but students only saw the lines instead of connecting them to the original function being studied. We were all frustrated. A few days later while walking down the craft aisle in a store, I stumbled on a feather boa that had haphazardly fallen onto the floor. The way it was situated, the boa resembled a large graph of a function, and I could imagine my students walking on it while physically exploring that analysis definition. Later in class, my students were surprised when I placed a feather boa on the classroom floor. Students volunteered to use yarn to represent the epsilon and delta regions around a point. They noticed how delta was affected by epsilon. The definition came alive. Throughout the rest of the course, we used the boa or a collection of boas whenever we covered new definitions about functions. Details for that activity can be found in *PRIMUS* [1].

After successfully using feather boas in analysis, I realized that the boas could also be used in any class that studies graphs of simple functions. I gathered a collection of feather boas and developed small group activities to aid students in understanding piecewise functions as well as activities for making connections between derivatives and the shape of graphs. The added beauty of using the boas in calculus and precalculus is that students typically are intrigued by the novelty and want to touch them; this increases student participation. Also, the graphs are large enough to see from the other side of the room, making it easy to tell which groups understand the material and which groups need a little more help. Since the boas worked well for small group activities, I brought them into my senior complex variables class where students used the boas to represent the images of a basic smiley face under a variety of standard complex functions. I never realized that feather boas were such a great teaching tool in mathematics. Even more, I never realized how an idea that worked in one class could be modified to create useful activities in a wide range of classes.

We have both realized the value of hands-on teaching and learning and have adopted it in a wide array of applications. Whether presenting a challenging concept or leveling the playing field for a diverse student population, we try to develop a meaningful hands-on activity that forces our students to engage with the material, ultimately developing their own concrete understanding. Of course, the joy of seeing a room full of engaged and excited students comes at a price, as these activities require planning and take time away from lecturing. Yet, activities do not have to be done at the expense of content. Lecture time is simply replaced with the activity, and the class then has a shared experience that can be referenced for many lessons down the road. Therefore, we believe that the benefits of using activities in class are worth the investment.

Each of the activities in this book has been used and vetted by its author(s), and the write-up includes suggestions and pitfalls to help reduce that initial investment of time. A more detailed explanation of the features of this book can be found in "How to Use This Book". We invite you to try some of the activities in this book, and we hope that you and your students both benefit from the deeper learning and the simple joy that hands-on activities can bring.

Sincerely,
Julie & Jessica

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How to Use This Book

Consider using this book in much the same way you would use a cookbook. Each hands-on learning activity is presented in recipe format with generally two pages describing the reasons, logistics, and helpful hints for running the activity, and another page that can be used as a handout in class.

These handouts are available free of charge at:
`www.ams.org/bookpages/clrm-54`.

The topics presented cover a variety of mathematical concepts found in courses at all levels. The activities are grouped by level. Activities designed for use in courses before calculus are presented as appetizers, as these courses provide an early introduction to the field of mathematics. Calculus activities are referred to as main courses, as the calculus sequence is a substantial portion of any mathematics curriculum. The book finishes with activities that are relevant to upper-level courses; these activities are called desserts given the joy that is often presented and discovered in these courses. Although the focus of this book is on courses taken by mathematics majors, several of the activities in this book can also be used in lower level exploratory topics courses, such as Math for Liberal Arts; an overview of such activities is presented on pages xvi-xx.

In terms of the time commitment needed to run these activities, they range from tiny morsels, short activities that require only a few minutes for implementation, to more hefty portions, long activities that would need to be savored for an entire class period.

As with any cookbook, it is possible to flip to an activity and follow the instructions exactly as presented. However, just as you might modify a recipe based on your taste or the availability of ingredients, we encourage you to modify the details of any activity to meet the specific needs of your students and constraints of your course. We also invite you to browse through the book simply for inspiration as you create and implement your own activity ideas.

We have included several indices to help you navigate the book.

Concept Index: This index makes it possible to search for activities related to a particular mathematical concept.

Author Index: This index can be used if you liked one activity and would like to see more by that author.

Main Ingredient Index: This index can be used to find an activity that uses a particular prop, like cookies, a sheet, or feather boas. It could also be used to see if this book includes an activity that describes mathematics using a random item you found on your shelf.

Course Index: This index identifies all the activities that could be used in a specific mathematics course. Activities may be listed under multiple courses.

We hope that this book will encourage and inspire you to explore the possibilities of using more hands-on activities in your classes. Bon appétit!

Modifications for Liberal Arts and Topics Courses

General education courses that explore a wide variety of mathematical topics, with titles like Mathematics for Liberal Arts, are becoming increasingly common. Many of the activities in this book can be used in such a course either as written or with modifications. Tables 1-6 give a brief overview of some topics and the activities that address them. Many of these activities do not require modification and are denoted by "-" in the corresponding comment column. Suggestions or modifications for all other activities are discussed in the numbered list below each table, where the numbers correlate to the notes column in the table.

Functions

Many lower level courses introduce students to functions. Four of these activities may require modifications that are given in the notes below. However, most of the activities listed in Table 1 on this topic can be used without modification and are designated with a "-". Regardless of whether modifications are discussed for a particular activity, we encourage you to think about your course and tailor that activity accordingly.

Table 1: Function activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
1.2	Function Ball Toss	-
5.13	Properties of Functions on Finite Sets Using Candy	1a
1.4	Function Composition Using Crackers and Cheese	-
1.5	Walking Function Transformations	-
1.1	Using Parenthesis with the Game of Telephone	-
1.6	Graphing Piecewise Functions with Feather Boas	-
4.2	Building Functions of Two Variables with Cookies	1b
4.3	Exploring Contours in the Physical World	1b
4.4	Matching Photographs with Contour Lines	1b

Notes:

1a. **Properties of Functions on Finite Sets Using Candy**: This activity, which can help introduce students to the definition of a function, was originally designed for an upper level course. As written, the activity has students engage with some of the more formal terminology, such as *injection*, *surjection*, *bijection*, *one-to-one*, and *onto*, and the handout uses this terminology as well as formal notation. Therefore, the handout may need to be altered to fit your course.

1b. **Building Functions of Two Variables with Cookies**, **Exploring Contours in the Physical World**, and **Matching Photographs with Contour Lines**: Each of these activities allows students to explore multivariable functions and methods of visualizing these functions at an introductory level. As these activities were originally designed for use in a multivariable calculus course, you may want to avoid using calculus terms in your explanations. On the handouts, you may opt to remove the formal domain notation used in **Cookies**; the other two handouts may be used as written.

Modeling

This book contains eight activities that relate to mathematical modeling, or the contextualization of mathematics. Although many of these activities were originally developed for the calculus level or higher, each can be made accessible to a broader audience through the suggestions given below.

Table 2: Modeling activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
1.3	Mathematical Modeling Using Long-exposure Photography	2a
2.1	Adding Movement to Velocity Explorations with Ziplines	2b
2.6	Spread the Word: Modeling Logistic Growth	2c
2.7	Maximizing the Area of a Fenced in Region Using...	2d
2.8	The Optimal Origami Box	2e
5.3	Population Modeling Using M&M's	2f
5.4	Modeling of Fishing and Restocking with Pennies	2f

Notes:

2a. **Mathematical Modeling Using Long-exposure Photography**: This activity is already targeted for a lower level course. If you wish to increase the accessibility further, remove the portion on computing error (Question 4 on the handout).

2b. **Adding Movement to Velocity Explorations with Ziplines**: This activity is designed to be used early in a calculus course as an exploratory introduction to the ideas of a limit and ultimately a derivative. As it is an introductory activity, the handout already avoids calculus terminology such as *limit* and *derivative*. Therefore, the handout for this activity can be used as is; we just caution you to avoid using calculus terminology in the discussion.

2c. **Spread the Word: Modeling Logistic Growth**: In this activity, students collect data on the spread of a disease amongst their classmates and do a fit to the data. Question 6 asks students to perform a logistic regression, which can be done using technology, avoiding the need for knowledge of this skill. Question 7 refers to an inflection point; if you choose to maintain the use of this terminology, you may need to define and discuss this with your students before they attempt that question.

2d. **Maximizing the Area of a Fenced in Region Using Bendable Sticks for Constraints**: Although this activity in optimizing a fenced area was originally written for a calculus course, most of the handout does not require any knowledge of calculus. Question 6 asks students to use calculus to find the optimal area, but this can be replaced with a graphical method without further modification.

2e. **The Optimal Origami Box**: This activity was originally designed for use in the optimization section of a calculus course, but a graphical optimization technique could be used instead. The handout is already written without reference to calculus ideas, as students are asked to develop their own approach to optimization. Therefore, the handout can be used as written, and you can simply direct student discussion towards a graphical approach.

2f. **Population Modeling Using M&M's** & **Modeling of Fishing and Restocking with Pennies**: Both of these activities are designed to motivate the study of differential equations through experiments in population dynamics. Although the instructor guidance for both drive these activities towards developing differential equations, the process of understanding the experiments, developing hypotheses, and testing these hypotheses are all still accessible without using a differential equation. Restricting the development to difference equations keeps the mathematics accessible, as these can be easily analyzed by hand or using a spreadsheet. The handout for **M&M's** is already written for a general audience, as it does not discuss differential equations. On the handout for **Fish**, you may wish to tailor Questions 3c and 4, as these questions currently deal with the formalization of the mathematical model, the solution process, and the assessment of the model.

Trigonometry

If your topics course includes a trigonometry component, then you may find two of the activities in this book helpful, one of which may require a minor modification.

Table 3: Trigonometry activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
1.7	Trigonometry Parameter Comparisons	-
4.12	Vector Analysis of Pop-Up Page	3a

Notes:

3a. **Vector Analysis of a Pop-Up Page**: This activity, which has been used in an introductory Math and Art course, provides an opportunity to apply trigonometry in three dimensional coordinate systems, which need to be taught prior to this activity. On the handout, Question 7 asks students to set up equations relating the positions of points of the pop-up. As this can be challenging, it is advisable that you demonstrate the first part of Question 7 at the board before groups attempt the rest of the question. It may even be necessary to walk the class through the entire problem, depending on the algebra skills of your students. All other portions of the activity should be accessible as long as your students understand algebraic manipulation and you've introduced basic trigonometry and three dimensional coordinate systems.

Logical Reasoning

Understanding logical arguments is an important and transferable skill, and as such, logical reasoning is a common topic in a general education topics course. All of the activities listed in the table can be used without modification, but we encourage you to look over the terminology and notation used on the handouts to make sure that you introduce these to your students as needed.

Table 4: Logic activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
5.8	Traffic Jam: A Lifesize Logic Puzzle	-
5.9	Living DeMorgan's Laws	-
5.10	Using Circuits to Teach Truth Tables	-
5.11	Determining the Validity of an Argument Using...	-
6.8	Walking the Seven Bridges of Konigsberg	-

Approximating Rates of Change, Areas, Volumes, and Series

Although most students in a general education level topics course will not take calculus, the ideas of calculus have broad applications, and it may be beneficial for all students, regardless of major, to have exposure to these ideas. Many of the exploratory calculus activities in this book can be easily modified to help all students understand the underpinning concepts of calculus without actually doing any calculus.

Table 5: Approximating rates of change, areas, volumes, and series activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
2.1	Adding Movement to Velocity Explorations with Ziplines	5a
3.1	Chewing Gum Riemann Sums	5b
3.2	Riemann Sums Using the Paper Shredder or Cut the Bunny	5b
3.3	Estimating Calories in a Cookie with Riemann Sums	5b
3.7	Fun with Infinite Series	5c
4.8	Volume Estimation Using a Sheet Surface	5d
4.9	Visualizing and Estimating the Mass of a Solid Using...	5d

Notes:

5a. **Adding Movement to Velocity Explorations with Ziplines**: This activity encourages students to think about how one might determine instantaneous rates of change based on experiments calculating average rates of change over a decreasing distance. This activity motivates the idea of a derivative without ever actually using that terminology. The handout can be used as written, and you should just be mindful of the terminology you use during the activity.5b. **Chewing Gum Riemann Sums**, **Riemann Sums Using the Paper Shred-**

der or Cut the Bunny, and **Estimating Calories in a Cookie with Riemann Sums**: Each of these activities has students explore area estimations using Riemann sums. None of the handouts use the terms relating to integration, so they may be used as written, unless you also wish to avoid the terminology of Riemann sums, in which case, the **Bunny** activity may be your preferred choice as it is very open-ended and does not use any formal language to guide the inquiry process.

5c. **Fun with Infinite Series**: This title actually covers three activities relating to infinite series, two of which are easily adapted for a lower level course. The **Fun with Paper** activity could be used without modification. The **Fun with Fractals** activity could be modified to remove the formal notation and calculation of series. Both of these activities can be used to reinforce algebraic notation and geometric sequences as students consider the sums and make hypotheses about whether the results are finite or infinite. The handouts are written in such a way that they can be used without modification. The **Fun with Gravity** activity could be similarly modified, but it may be more challenging for students and requires some understanding of physics.

5d. **Volume Estimation Using a Sheet Surface** and **Visualizing and Estimating the Mass of a Solid Using Multi-colored Blocks**: These two activities have students explore simple approximations for the volume and mass, respectively, of a three dimensional object. The **Sheet** activity handout can be used as written, as it does not use any formal terminology or notation. You may wish to modify the handout for the **Blocks** activity, as it uses formal notation.

Exposure to Higher Mathematics

Several topics courses are designed to introduce students to some interesting ideas found in higher level mathematics that they would otherwise not encounter in their college careers as non-mathematics majors. The activities listed in this section of the table offer opportunities to support this type of course. Many of these activities do not require modification; those that require modification are discussed below. For all activities, we encourage you to be mindful of the terminology and notation you introduce and use in your course.

Table 6: Higher level mathematics activities that would work in a liberal arts course.

Activity Number	Activity Name	See Note Number
6.8	Walking the Seven Bridges of Königsberg	-
6.9	Designing Round - Robin Tournaments Using Yarn	-
6.15	Exploring Knots	-
6.1	Using Candy to Represent Equivalence Relations	-
5.7	Picturing Prime Factorization	-
6.7	Discovering Catalan Numbers Using M&M's	-
6.3	Symmetry and Group Theory With Plastic Triangles	6a
6.5	Acting Permutations	6b
5.14	SET in Combinatorics and Discrete Math	6c

Notes:

6a. **Symmetry and Group Theory With Plastic Triangles**: This activity, which focuses on identifying groups, can be used in a topics course to demonstrate an abstract side of mathematics that highlights the field as one of the liberal arts. The questions on the handout get at some deeper concepts, including identities and inverses, which could be excluded; however, much of this activity, including the building of the group table has been completed by children in the fourth grade.

6b. **Acting Permutations**: This activity has been used in both mathematics education and mathematics for liberal arts courses by modifying the activity to limit the scope of the explorations to permutations, inverses, commutativity, identity, and order. To add meaning to the activity, you can emphasize examples of permutations in the real world and the thought process behind generating and testing conjectures.

6c. **SET in Combinatorics and Discrete Math**: This activity can be used in a wide variety of settings, as long as you introduce your students to combinations and the multiplication principle. Students in a lower level course may have difficulty with Question 2 on the handout, so you may consider removing it or providing students with more guidance than you would in an upper level mathematics course.

Concept Index

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Course Index

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Q: What do feather boas, cookies, and paper shredders have in common?

A: They are all ingredients that have the potential to help your undergraduate students understand a variety of mathematical concepts. In this book, 43 faculty from a wide range of institutional settings share a total of 64 hands-on activities that allow students to physically engage with mathematical ideas ranging from the basics of precalculus to special topics appropriate for upper-level courses. Each learning activity is presented in an easy-to-read recipe format that includes a list of supplies; a narrative briefly describing the reasons, logistics, and helpful hints for running the activity; and a page that can be used as a handout in class. Purchase of the book also includes access to electronic printable versions of the handouts.

With so many activities, it might be hard to decide where to start. For that reason, there are four indices to help the reader navigate this book: a concept index, a course index, an author index, and a main ingredient index. In addition to providing activities for precalculus, calculus, commonly required mathematics courses for majors, and more specialized upper-level electives, there is also a section describing how to modify many of the activities to fit into a liberal arts mathematics class.

Whether you are new to using hands-on activities in class or are more experienced, the authors hope that this book will encourage and inspire you to explore the possibilities of using more hands-on activities in your classes.

Bon appétit!

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