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TO F. D. A.

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PREFACE

The theory of linear associative algebras probably reached its zenith when the solution was found for the problem of determining all rational division algebras. Since that time it has been my hope that I might develop a reasonably self-contained exposition of that solution as well as of the theory of algebras upon which it depends and which contains the major portion of my own discoveries. The first step in carrying out this desire was necessarily that of writing a text with contents selected so as to provide a foundation adequate for the prospective exposition. This text has already been published under the title *Modern Higher Algebra.* Its completion was followed shortly by the timely invitation of the Colloquium Committee of the American Mathematical Society to me to write these LECTURES embodying the desired exposition.

It has been most fortunately possible at this time to give a new treatment of the early parts of our subject simplifying not only the proofs in the theory of normal simple algebras but even the exposition of the structure theorems of Wedderburn. This does not evidence itself in the somewhat classical first two chapters. These contain the preliminary discussion of linear sets, direct products, direct sums, ideals in an algebra, and similar topics, with the additions and modifications made necessary by the fact that we are considering here algebras over an arbitrary field and that inseparable extension fields may exist. But the exposition given of the usual fundamental theorem, stating that every linear associative algebra is equivalent to a first algebra of square matrices and reciprocal to a second such algebra, is expanded here so as to have as consequence a result basic in the new treatment of the Wedderburn structure theory. While this result is not derived until its need appears in Chapter III the proof is so elementary that it might have been placed in the first chapter without change.

This basic theorem is that of R. Brauer on the structure of the direct product of a normal division algebra and its reciprocal algebra. It is combined with two theorems of J. H. M. Wedderburn to obtain as generalizations three tool theorems which are used throughout Chapters III and IV, and yield rather remarkable simplifications of the proofs of numerous fundamental results. In particular in Chapter III the foundation of the proof of the Wedderburn principal theorem on the structure of an algebra with a radical is simplified and the theorem itself then obtained.

While the first three chapters of this exposition do contain a considerable amount of new material their principal content is evidently an exposition in more modern form of the Wedderburn structure theorems which were first presented in book form in the two editions of the text on our subject by L. E. Dickson. I owe much to these expositions as well as to their author, who has been my teacher and the inspiration of all my research. The remaining eight

chapters of the present text are composed principally of results derived since 1926 when the second (German) edition of Dickson's text was written. These results are due principally to R. Brauer, H. Hasse, E. Noether, and myself. Their exposition is begun in Chapter IV which contains the theory of the commutator subalgebra of a simple subalgebra of a normal simple algebra, the study of automorphisms of a simple algebra, splitting fields, and the index reduction factor theory.

The fifth chapter contains the foundation of the theory of crossed products and of their special case, cyclic algebras. The theory of exponents is derived there as well as the consequent factorization of normal division algebras into direct factors of prime-power degree.

Chapter VI consists of the recent study of the abelian group of cyclic systems which is applied in Chapter VII to yield the theory of the structure of direct products of cyclic algebras and the consequent properties of norms in cyclic fields. This chapter is closed with the recently developed theory of p-algebras.

In Chapter VIII an exposition is given of the theory of the representations of algebras. The treatment is somewhat novel in that while the recent expositions have used representation theorems to obtain a number of results on algebras, here the theorems on algebras are themselves used in the derivation of results on representations. The presentation has its inspiration in my work on the theory of Riemann matrices and is concluded by an introduction to the generalization (by H. Weyl and myself) of that theory.

In the ninth chapter the structure of rational division algebras is determined. The study begins with a detailed exposition of Hasse's theory of p-adic division algebras. The results are then extended so as to yield the theorems on rational division algebras without recourse to the theory of ideals in the integral sets of such algebras. This is believed to be the first time the extension has been made in a really simple fashion. The method is a greatly desirable one as the previous treatments used the results of a very voluminous theory which we are able to omit. It is necessary, of course, to assume without proof certain existence theorems from the theory of algebraic numbers. These presupposed theorems are indicated precisely, and I hope to be able to include their proofs in a future text on the theory of algebraic numbers and the arithmetic of algebras.

The theory of involutorial simple algebras arose in connection with the study of Riemann matrices but is now a separate branch of the theory of simple algebras with structure theorems on approximately the same level as those on arbitrary simple algebras. This theory is derived in Chapter X both for algebras over general fields and over the rational field. The results are also applied in the determination of the structure of the multiplication algebras of all generalized Riemann matrices, a result which is seen in Chapter XI to imply a complete solution of the principal problem on Riemann matrices.

This final reference is but one item in the last chapter which contains an exposition of a number of special results. In particular there are given new

derivations of the structure of all normal division algebras of degrees three and four over any field. References to sources for the whole text are given in this chapter as well as indications of the literature on the subject and an extensive bibliography.

It is my hope that the form of this exposition will make it useful as a text on the theory of linear associative algebras as well as for its obvious purpose as a source book for young algebraists. Much of any success that there may have been in keeping the exposition completely correct and clear is due to the work of Dr. Sam Perlis who read the manuscript critically in each of the stages of its preparation. He not only assisted in keeping the exposition free of error but frequently indicated improvements resulting in greater clarity. I give him my great thanks.

I appreciate also the kind assistance of Professor Nathan Jacobson who suggested the proofs of two of the theorems as well as that of Mr. Morris Bloom who assisted in the preparation of the bibliography, and give thanks to Professor Saunders MacLane and Dr. Otto F. G. Schilling who were a critical audience for oral expositions of some of the proofs. Final thanks are due to Professor G. A. Bliss without whose encouragement the completion of these LECTURES would have been greatly delayed.

A. A. ALBERT

THE UNIVERSITY OF CHICAGO
March 5, 1939

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BIBLIOGRAPHY

Acc. d'It.—Accademia d'Italia. Rome. Memorie.
Atti Nap.—R. Accademia delle Scienze. Naples. Atti.
Atti Tor.—R. Accademia delle Scienze di Torino. Atti.
Atti Lin.—Accademia Nazionale dei Lincei. Atti.
Acta Arith.—Acta Arithmetica.
Act. Sci. Ind. Paris—Actualités Scientifiques et Industrielles. Paris.
Ak. Berlin S. B.—K. Akademie der Wissenschaften. Berlin. Sitzungsberichte.
Ak. Vienna S. B.—K. Akademie der Wissenschaften. Vienna. Sitzungsberichte.
Am. J.—American Journal of Mathematics.
A. M. S. Bull.—American Mathematical Society. Bulletin.
A. M .S. Coll.—American Mathematical Society. Colloquium Publications.
A. M. S. Trans.—American Mathematical Society. Transactions.
Ann. di Mat.—Annali di Matematica Pura ed Applicata.
Ann. of Math.—Annals of Mathematics.
Bull. Aca. Sci. U. S. S. R.—Bulletin of the Academy of Sciences. U. S. S. R.
Calc. M. S. Bull.—Calcutta Mathematical Society. Bulletin.
Camb. Ph. S. Pro.—Cambridge Philosophical Society. Proceedings.
Cir. Mat. Pal.—Circolo Matematico di Palermo. Rendiconti.
Chi. M. S. J.—Chinese Mathematical Society. Journal.
Comm. Math. Helv.—Commentarii Mathematici Helvetici.
Comp. Math.—Compositio Mathematica.
C. R. Paris.—Institut de France. Académie des Sciences. Comptes Rendus.
Deut. Math.—Deutsche Mathematik.
D. M. V. Jahr.—Deutsche Mathematiker-Vereinigung. Jahresbericht.
Duke J.—Duke Mathematical Journal.
Edin. M. S. Pro.—Edinburgh Mathematical Society. Proceedings.
Erg. der Math.—Ergebnisse der Mathematik.
Fund. Math.—Fundamenta Mathematicae.
Giornale—Giornale di Matematiche.
Gött. Nach.—Gesellschaft der Wissenschaften zu Göttingen. Nachrichten.
Hamb. Abh.—Hamburg Mathematisches Seminar. Abhandlungen.
Hans. Abh.—Hansische Universität. Mathematisches Seminar. Abhandlungen.
Hiro. J.—Hiroshima Imperial University, Japan. Journal of the Faculty of Sciences.
Hokk. J.—Hokkaido Imperial University, Japan. Mathematical Society. Journal.
Imp. Aca. Tok. Pro.—Imperial Academy, Tokyo. Proceedings.
Ind. Aca. Pro.—Indian Academy of Sciences. Proceedings.
Ind. M. S. J.—Indian Mathematical Society. Journal.
Jap. J.—Japanese Journal of Mathematics.
Jassy Ann.—Jassy, Roumania. Universitatea. Annales Scientifiques.
J. de Math.—Journal de Mathématiques.
J. für Math.—Journal für die reine und angewandte Mathematik.
London M. S. J.—London Mathematical Society. Journal.
London M. S. Pro.—London Mathematical Society. Proceedings.
Math. Ann.—Mathematische Annalen.
Math. Zeit.—Mathematische Zeitschrift.
Mess. Math.—Messenger of Mathematics.
Monat.—Monatshefte für Mathematik und Physik.
P. N. A. S.—National Academy of Sciences. Proceedings.

N. R. C. Bull.—National Research Council. Bulletin.
Nat. Ges. Zur. V.—Naturforschende Gesellschaft, Zurich. Vierteljahreschrift.
P.-M. S. Jap.—Physico-Mathematical Society of Japan. Proceedings.
Rome Rend.—Rome Università. Seminario Matematico. Rendiconti.
R. S. Can. Trans.—Royal Society of Canada. Transactions.
R. S. Edin. Pro.—Royal Society of Edinburgh. Proceedings.
Toh. Imp. U. Sci. Rep.—Science Reports of the Tôhoku Imperial University. Sendai, Japan.
Schw. Nat. Ges. V.—Schweizerische Naturforschende Gesellschaft. Verhandlungen.
Soc. Pol. Mat. Ann.—Société Polonaise de Mathématiques. Annales.
Tôhoku J.—Tôhoku Mathematical Journal.
Tokyo J.—Tokyo. Faculty of Science. Journal.
Toronto St.—Toronto University. Studies.
Zeit. Phy.—Zeitschrift für Physik.
Zentralblatt—Zentralblatt für Mathematik und ihre Grenzgebiete.

AKIZUKI, Y. [1]: *Eine homomorphe Zuordnung der Elemente der galoisschen Gruppe zu den Elementen einer Untergruppe der Normklassengruppe*, Math. Ann., v. 112 (1936), 566-71.

ALBERT, A. A. [2]: *A determination of all normal division algebras in* 16 *units*, A. M. S. Trans., v. 31 (1929), 253-60; [3]: *On the rank equation of any normal division algebra*, A. M. S. Bull., v. 35 (1929), 335-8; [4]: *The rank function of any simple algebra*, P. N. A. S., v. 15 (1929), 272-6; [5]: *On the structure of normal division algebras*, Ann. of Math., v. 30 (1929), 322-38; [6]: *Normal division algebras in* $4p^2$ *units, p an odd prime*, Ann. of Math., v. 30 (1929), 583-90; [7]: *Direct products of rational quaternion algebras*, Ann. of Math., v. 30 (1929), 621-5; [8]: *The non-existence of pure Riemann matrices with normal multiplication algebras of order* 16, Ann. of Math., v. 31 (1930), 375-80; [9]: *A necessary and sufficient condition for the equivalence of rational generalized quaternion algebras*, A. M. S. Bull., v. 36 (1930), 535-40; [10]: *Determination of all normal division algebras in* 36 *units of type* R_2, Am. J., v. 52 (1930), 283-92; [11]: *A note on an important theorem on normal division algebras*, A. M. S. Bull., v. 36 (1930), 649-50; [12]: *New results in the theory of normal division algebras*, A. M. S. Trans., v. 32 (1930), 171-95; [13]: *A construction of all non-commutative rational division algebras of degree* 8, Ann. of Math., v. 31 (1930), 567-76; [14]: *The structure of pure Riemann matrices with non-commutative multiplication algebras*, Cir. Mat. Pal., v. 55 (1931), 57-115; [15]: *On direct products, cyclic division algebras, and pure Riemann matrices*, A. M. S. Trans., v. 33 (1931), 219-34; [16]: *On normal division algebras of type R in* 36 *units*, A. M. S. Trans., v. 33 (1931), 235-43; [17]: *On the Wedderburn norm condition for cyclic algebras*, A. M. S. Bull, v. 37 (1931), 301-12; [18]: *A note on cyclic algebras of order* 16, A. M. S. Bull., v. 37 (1931), 727-30; [19]: *On direct products*, A. M. S. Trans., v. 33 (1931), 620-5; [20]: *Division algebras over an algebraic field*, A. M. S. Bull., v. 37 (1931), 777-84; [21]: *The structure of matrices with any normal division algebra of multiplications*, Ann. of Math., v. 32 (1931), 131-48; [22]: *On the construction of cyclic algebras with a given exponent*, Am. J., v. 54 (1932), 1-13; [23]: *Algebras of degree* 2^e *and pure Riemann matrices*, Ann. of Math., v. 33 (1932), 311-8; [24]: *A construction of non-cyclic normal division algebras*, A. M. S. Bull., v. 38 (1932), 449-56; [25]: *Normal division algebras of degree four over an algebraic field*, A. M. S. Trans., v. 34 (1932), 363-72; [26]: *A note on normal division algebras of order sixteen*, A. M. S. Bull., v. 38 (1932), 703-6; [27]: *On normal simple algebras*, A. M. S. Trans., v. 34 (1932), 620-5; [28]: *A note on the equivalence of algebras of degree* 2, A. M. S. Bull., v. 39 (1933), 257-8; [29]: *On primary normal division algebras of degree eight*, A. M. S. Bull., v. 39 (1933), 265-72; [30]: *Non-cyclic algebras of degree and exponent four*, A.M.S. Trans., v. 35 (1933), 112-21; [31]: *A note on the Dickson theorem on universal ternaries*, A. M. S. Bull., v. 39 (1933), 585-8; [32]: *Normal division algebras over an algebraic number field not of finite degree*, A. M. S. Bull., v. 39 (1933), 746-9; [33]: *On the construction of Riemann matrices*, I, Ann. of Math., v. 35 (1934), 1-28; [34]: *On a certain algebra of*

quantum mechanics, Ann. of Math., v. 35 (1934), 65–73; [**35**]: *Normal division algebras of degree four over F of characteristic two*, Am. J., v. 56 (1934), 75–86; [**36**]: *Integral domains of generalized quaternion algebras*, A. M. S. Bull., v. 40 (1934), 164–76; [**37**]: *Normal division algebras over a modular field*, A. M. S. Trans., v. 36 (1934), 388–94; [**38**]: *A solution of the principal problem in the theory of Riemann matrices*, Ann. of Math., v. 35 (1934), 500–15; [**39**]: *On normal Kummer fields over a non-modular field*, A. M. S. Trans., v. 36 (1934), 885–92; [**40**]: *The principal matrices of a Riemann matrix*, A. M. S. Bull.,v. 40 (1934), 843–6; [**41**]: *A note on the Poincaré theorem on impure Riemann matrices*, Ann. of Math., v. 36 (1935), 151–6; [**42**]: *On the construction of Riemann matrices*, II, Ann. of Math., v. 36 (1935), 376–94; [**43**]: *Involutorial simple algebras and real Riemann matrices*, Ann. of Math., v. 36 (1935), 886–964; [**44**]: *Normal division algebras of degree p^e over F of characteristic p*, A. M. S. Trans., v. 39 (1936), 183–8; [**45**]: *Simple algebras of degree p^e over a centrum of characteristic p*, A. M. S. Trans., v. 40 (1936), 112–26; [**46**]: *p-algebras over a field generated by one indeterminate*, A. M. S. Bull., v. 43 (1937), 733–6; [**47**]: *Modern Higher Algebra*, Chicago, 1937; [**48**]: *Non-cyclic algebras with pure maximal subfields*, A. M. S. Bull., v. 44 (1938), 576–9; [**49**]: *Quadratic null forms over a function field*, Ann. of Math., v. 39 (1938), 494–505; [**50**]: *On cyclic algebras*, Ann. of Math., v. 39 (1938), 669–82; [**51**]: *A note on normal division algebras of prime degree*, A. M. S. Bull., v. 44 (1938), 649–52.

ALBERT, A. A., and HASSE, H. [**52**]: *A determination of all normal division algebras over an algebraic number field*, A. M. S. Trans., v. 34 (1932), 722–6.

ARCHIBALD, R. G. [**53**]: *Diophantine equations in division algebras*, A. M. S. Trans., v. 30 (1928), 819–37.

ARTIN, E. [**54**]: *Über einen Satz von Herrn J. H. M. Wedderburn*, Hamb. Abh., v. 5 (1928), 245–50; [**55**]: *Zur Theorie der hyperkomplexen Zahlen*, ibid., 251–60; [**56**]: *Zur Arithmetik hypercomplexer Zahlen*, loc. cit., 261–89.

ARTIN, E., and SCHREIER, O. [**57**]: *Algebraische Konstruktion reeler Körper*, Hamb. Abh., v. 5 (1928), 83–115; [**58**]: *Eine Kennzeichnung der reell abgeschlossenen Körper*, ibid. 225–31.

ASANO, K. [**59**]: *Über die Darstellungen einer endlichen Gruppe durch reele Kollineationen*, Imp. Aca. Tok. Pro., v. 9 (1933), 574–6; [**60**]: *Zur Diskriminante einer Algebra*, Jap. J., v. 12 (1935), 51–8.

ASANO, K., and NAKAYAMA, T. [**61**]: *Über halblinear Transformationen*, Math. Zeit., v. 115 (1937), 87–114.

ASANO, K., and SHODA, K. [**62**]: *Zur Theorie der Darstellungen einer endlichen Gruppe durch Kollineationen*, Comp. Math., v. 2 (1935), 230–40.

BIRKHOFF, G. [**63**]: *On the combination of subalgebras*, Camb. Ph. S. Pro., v. 29 (1933), 441–64; [**64**]: *Note on the paper "On the combination of subalgebras,"* Camb. Ph. S. Pro., v. 30 (1934), 200; [**65**]: *Ideals in algebraic rings*, P. N. A. S., v. 20 (1934), 571–3; [**66**]: *On the structure of abstract algebras*, Camb. Ph. S. Pro., v. 31 (1935), 433–54; [**67**]: *Representability of Lie algebras and Lie groups by matrices*, Ann. of Math., v. 38 (1937), 526–32.

BOYCE, F. [**68**]: *Certain types of nilpotent algebras*, Chicago dissertation, 1938.

BRANDT, H. [**69**]: *Zur allgemeinen Idealtheorie*, Schw. Nat. Ges. V., v. 26 (1927), 318–19; [**70**]: *Idealtheorie in Quaternionenalgebren*, Math. Ann., v. 99 (1928), 1–29; [**71**]: *Idealtheorie in einer Dedekindschen Algebra*, D. M. V. Jahr., v. 37 (1928), 5–7; [**72**]: *Primidealzerlegung in einer Dedekindschen Algebra*, Schw. Nat. Ges. V., v. 28 (1929), 288–90; [**73**]: *Zur Idealtheorie Dedekindscher Algebren*, Comm. Math. Helv., v. 2 (1930), 13–17.

BRAUER, R. [**74**]: *Über Zusammenhänge zwischen arithmetischen und invariantentheoretischen Eigenschaften von Gruppen linearer Substitutionen*, Ak. Berlin S. B., v. 30 (1926), 410–6; [**75**]: *Über die Darstellung der Drehungsgruppe durch Gruppen linearer Substitutionen*, Berlin dissertation, 1925; [**76**]: *Untersuchungen über die arithmetischen Eigenschaften von Gruppen linearer Substitutionen*, Math. Zeit., v. 28 (1928), 677–96; [**77**]: ibid., II, v. 31 (1930), 733–47; [**78**]: *Über Systeme hyperkomplexer Zahlen*, Math. Zeit., v. 29 (1929),

79–107; [79]: *Die stetigen Darstellungen der complexen orthogonalen Gruppe*, Ak. Berlin S. B., v. 29 (1929), 626–38; [80]: *Über die algebraische Struktur von Schiefkörpern*, J. für Math., v. 166 (1932), 241–52; [81]: *Über die Konstruktion der Schiefkörper, die von endlichem Rang in bezug auf ein gegebenes Zentrum sind*, J. für Math., v. 168 (1932), 44–64; [82]: *Über den Index und den Exponenten von Divisionsalgebren*, Tôhoku J., v. 37 (1933), 77–87; [83]: *Über die Kleinsche Theorie der algebraischen Gleichungen*, Math. Ann., v. 110 (1934), 473–500; [84]: *Über die Darstellung von Gruppen in Galoisschen Feldern*, Act. Sci. Ind. Paris, 1935, 15 pp.; [85]: *Eine Bedingung für vollständige Reduzibilität von Darstellungen gewöhnlicher und infinitesimaler Gruppen*, Math. Zeit., v. 41 (1936), 330–9; [86]: *On algebras which are connected with the semi-simple continuous groups*, Ann. of Math., v. 38 (1937), 857–72; [87]: *Sur la multiplication des caractéristiques des groups continues et semi-simples*, C. R. Paris, v. 204 (1937), 1784–6; [88]: *On normal division algebras of index five*, P. N. A. S., v. 24 (1938), 243–6.

BRAUER, R., HASSE, H., and NOETHER, E. [89]: *Beweis eines Hauptsatzes in der Theorie der Algebren*, J. für Math., v. 167 (1931), 399–404.

BRAUER, R., and NESBITT, C. [90]: *On the regular representations of algebras*, P. N. A. S., v. 23 (1937), 236–40; [91]: *On the modular representation of groups of finite order*, Toronto St., 1938, 21 pp.

BRAUER, R., and NOETHER, E. [92]: *Über minimale Zerfällungskörper irreduzibler Darstellungen*, Ak. Berlin S. B., v. 32 (1927), 221–6.

BRAUER, R., and SCHUR, I. [93]: *Zum Irreduzibilitätsbegriff in der Theorie der Gruppen linearer homogener Substitutionen*, Ak. Berlin S. B., v. 34 (1930), 209–26.

BUSH, L. E. [94]: *Note on the discriminant matrix of an algebra*, A. M. S. Bull., v. 38 (1932), 49–51; [95]: *On Young's definition of an algebra*, A. M. S. Bull., v. 39 (1933), 142–8.

CASIMIR, H., and VAN DER WAERDEN, B. L. [96]: *Algebraischer Beweis der vollständigen Reduzibilität der Darstellungen halbeinfacher Liescher Gruppen*, Math. Ann., v. 111 (1935), 1–12.

CECIONI, F. [97]: *Sopra un tipo di algebre prive di divisori dello zero*, Cir. Mat. Pal., v. 47 (1923), 209–54.

CHERUBINO, S. [98]: *Sulle varietà abeliane reale, e sulle matrici di Riemann reali*, I, Giornale, v. 60 (1922), 65–94; [99]: Ibid., II, v. 61 (1923), 47–68; [100]: *Sulle serie di potenze di una variabile, in un algebra*, Atti Lin., v. 22 (1935), 211–16.

CHEVALLEY, C. [101]: *Sur un théorème de M. Hasse*, C. R. Paris, v. 191 (1930), 369–70; [102]: *Sur la théorie des restes normiques*, C. R. Paris, v. 191 (1930), 426–8; [103]: *Sur la structure de la théorie du corps de classes*, C. R. Paris, v. 194 (1932), 766–9; [104]: *Sur la théorie du corps de classes*, Paris dissertation, 1932; [105]: *La théorie du symbole de restes normiques*, J. für Math., v. 169 (1932), 141–57; [106]: *Sur la théorie du corps de classes dans les corps finis et les corps locaux*, Tokyo J., v. 2 (1933), 365–474; [107]: *Sur certains idéaux d'une algèbre simple*, Hamb. Abh., v. 10 (1934), 83–105; [108]: *Démonstration d'une hypothèse de M. Artin*, Hamb. Abh., v. 11 (1935), 73–5; [109]: *Sur la théorie du corps de classes*, C. R. Paris, v. 210 (1935), 632–4; [110]: *L'arithmétique dans les algèbres de matrices*, Act. Sci. Ind. Paris, 323, 1936, 35 pp.

COMESSATTI, A. [111]: *Fondamenti per la geometria sopra le superficie razionali dal punto di vista reale*, Math. Ann., v. 73 (1913), 1–72; [112]: *Sulle varietà abeliane reali* I, Ann. di Mat., v. 2 (1935), 67–106; [113]: Ibid. II, v. 3 (1926), 27–71; [114]: *Intorno ad un nuovo carattere delle matrici di Riemann*, Acc. d'It., v. 7 (1936), 81–129.

CONWELL, H. H. [115]: *Linear associative algebras of infinite order whose elements satisfy finite algebraic equations*, A. M. S. Bull., v. 40 (1934), 95–102.

DARKOW, M. D. [116]: *Determination of a basis for the integral elements of certain generalized quaternion algebras*, Ann. of Math., v. 26 (1926), 263–70.

DEURING, M. [117]: *Zur Theorie der Normen relativzyklischer Körper*, Gött. Nach., (1931), 199–200; [118]: *Galoissche Theorie und Darstellungstheorie*, Math. Ann., v. 107 (1932), 140–4; [119]: *Algebren*, Erg. der Math., v. 4, 1935, 143 pp.; [120]: *Über den Hauptsatz*

der Algebrentheorie, J. für Math., v. 175 (1936), 63–4; [**121**]: *Einbettung von Algebren in Algebren mit kleinerem Zentrum*, J. für Math., v. 175 (1936), 124–8.

DICKSON, L. E. [**122**]: *Definitions of a linear associative algebra by independent postulates*, A. M. S. Trans., v. 4 (1903), 21–6; [**123**]: *On hypercomplex number systems*, A. M. S. Trans., v. 6 (1905), 344–8; [**124**]: *On finite algebras*, Gött. Nach., (1905), 358–93; [**125**]: *Linear algebras in which division is always uniquely possible*, A. M. S. Trans., v. 7 (1906), 370–90; [**126**]: *On commutative linear algebras in which division is always uniquely possible*, A. M. S. Trans., v. 7 (1906), 514–22; [**127**]: *On triple algebras and ternary cubic forms*, A. M. S. Bull., v. 14 (1908), 160–9; [**128**]: *Linear algebras*, A. M. S. Trans., v. 13 (1912), 59–73; [**129**]: *Linear algebras*, Cambridge, 1914, 73 pp.; [**130**]: *Linear associative algebras and abelian equations*, A. M. S. Trans., v. 15 (1914), 31–46; [**131**]: *On the relation between linear algebras and continuous groups*, A. M. S. Bull., v. 22 (1915), 53–61; [**132**]: *On quaternions and their generalization and the history of the eight square theorem*, Ann. of Math., v. 20 (1919), 155–71; [**133**]: *Arithmetic of quaternions*, London M. S. J., v. 20 (1931), 225–32; [**134**]: *Quaternions and their generalizations*, Mess. Math., v. 7 (1921), 109–14; [**135**]: *Impossibility of restoring unique factorization in a hypercomplex arithmetic*, A. M. S. Bull., v. 28 (1922), 438–42; [**136**]: *Algebras and their Arithmetics*, Chicago, 1923, 241 pp.; [**137**]: *The rational linear algebras of maximum and minimum ranks*, London M. S. Pro., v. 22 (1923), 145–62; [**138**]: *A new simple theory of hypercomplex integers*, J. de Math., v. 2 (1923), 281–326; [**139**]: *Algebras and their arithmetics*, A. M. S. Bull., v. 30 (1924), 247–57; [**140**]: *On the theory of numbers and generalized quaternions*, Am. J., v. 46 (1924), 1–16; [**141**]: *New division algebras*, A. M. S. Trans., v. 28 (1926), 207–34; [**142**]: *Algebren und ihre Zahlentheorie*, Zurich, 1927, 308 pp.; [**143**]: *New division algebras*, A. M. S. Bull., v. 34 (1928), 555–60; [**144**]: *Further development of the theory of arithmetics of algebras*, Toronto Congress Proceedings (1928), 173–84; [**145**]: *Construction of division algebras*, A. M. S. Trans., v. 32 (1930), 319–34; [**146**]: *Linear algebras with associativity not assumed*, Duke J., v. 1 (1935), 113–25.

DORROH, J. L. [**147**]: *Concerning adjunctions to algebras*, A. M. S. Bull., v. 38 (1932), 85–8.

EICHLER, M. [**148**]: *Untersuchungen in der Zahlentheorie der rationalen Quaternionenalgebren*, J. für Math., v. 174 (1936), 129–59; [**149**]: *Über die Idealklassenzahl total definiter Quaternionenalgebren*, Math. Zeit., v. 43 (1937), 102–9; [**150**]: *Bestimmung der Idealklassenzahl in gewissen normalen einfachen Algebren*, J. für Math., v. 176 (1937), 192–202; [**151**]: *Über die Einheiten der Divisionsalgebren*, Math. Ann., v. 114 (1937), 635–54; [**152**]: *Über die Idealklassenzahl hypercomplexer Systeme*, Math. Zeit., v. 43 (1938), 481–94.

FINAN, E. J. [**153**]: *A determination of the domains of integrity of the complete matric algebra of order* 4, Am. J., v. 53 (1931), 920–8; [**154**]: *On the number theory of certain non-maximal domains of the total matric algebra of order* 4, Duke J., v. 1 (1935), 484–90.

FITTING, H. [**155**]: *Die Theorie der Automorphismenringe Abelscher Gruppen und ihr Analogon bei nicht kommutativen Gruppen*, Math. Ann., v. 107 (1932), 514–42; [**156**]: *Der Normenbegriff für die Ideale eines Ringes beliebiger Struktur*, J. für Math., v. 178 (1937), 107–22.

FUETER, R. [**157**]: *Reziprozitätsgesetze in quadratisch-imaginären Körpern*, Gött. Nach., (1927), 427–45; [**158**]: *Über eine speziale Algebra*, J. für. Math., v. 167 (1931), 52–61; [**159**]: *Analytische Funktionen einer Quaternionenvariablen*, Comm. Math. Helv., v. 4 (1932), 9–20; [**160**]: *Formes d'Hermite, groupe de Picard et théorie des ideaux de quaternions*, C. R. Paris, v. 194 (1932), 2009–11; [**161**]: *Quaternionenringe*, Comm. Math. Helv., v. 6 (1934), 199–222; [**162**]: *Zur Theorie der Brandtschen Quaternionenalgebren*, Math. Ann., v. 110 (1934), 650–61; [**163**]: *Über die analytische Darstellung der regulären Funktionen einer Quaternionenvariablen*, Comm. Math. Helv., v. 8 (1936), 371–8; [**164**]: *Die Theorie der regulären Funktionen einer Quaternionenvariablen*, Oslo Congress Proceedings, 1937, 75–91.

FURTWANGLER, P., and TAUSSKY, O. [**165**]: *Über Schiefringe*, Ak. Vienna S. B., v. 7 (1936), 38.

GARVER, R. [166]: *Division algebras of order sixteen*, Ann. of Math., v. 28 (1927), 493–500.

GHENT, K. S. [167]: *A note on nilpotent algebras in four units*, A. M. S. Bull., v. 40 (1934), 331–8.

GRELL, H. [168]: *Zur Normentheorie in hyperkomplexen Systemen*, J. für Math., v. 162 (1930), 60–62.

GRIFFITHS, L. W. [169]: *Generalized quaternion algebras and the theory of numbers*, Am. J., v. 50 (1928), 303–14.

GRUNWALD, W. [170]: *Ein allgemeines Existenztheorem für algebraische Zahlkörper*, J. für Math., v. 169 (1933), 103–7.

HAANTJES, J. [171]: *Halblineare Transformationen*, Math. Ann., v. 114 (1937), 293–304.

HASSE, H. [172]: *Darstellbarkeit von Zahlen durch quadratische Formen in einem beliebigen algebraischen Zahlkörper*, J. für Math., v. 152 (1923), 113–30; [173]: *Äquivalenz quadratischer Formen in einem beliebigen algebraischen Zahlkörper*, J. für Math., v. 153 (1924), 158–62; [174]: *Bericht über neuere Untersuchungen und Probleme der algebraischen Zahlkörper*, D. M. V. Jahr. Supplementary v. 6 (1930), p. 38; [175]: *Neue Begründung und Verallgemeinerung der Theorie des Normenrestsymbols*, J. für Math., v. 162 (1930), 134–44; [176]: *Die Normenresttheorie relativ-Abelscher Zahlkörper als Klassenkörpertheorie im Kleinen*, J. für Math., v. 162 (1930), 145–54; [177]: *Über $\mathfrak{P}$-adische Schiefkörper und ihre Bedeutung für die Arithmetik hyperkomplexer Zahlkörper*, Math. Ann., v. 104 (1931), 495–534; [178]: *Beweis eines Satzes und Widerlegung einer Vermutung über das allgemeine Normenrestsymbol*, Gött. Nach., (1931), 64–9; [179]: *Theory of cyclic algebras over an algebraic number field*, A. M. S. Trans., v. 34 (1932), 171–214; [180]: *Additional note to the author's "Theory of cyclic algebras over an algebraic number field,"* ibid., 727–30; [181]: *Die Struktur der R. Brauerschen Algebrenklassengruppe über einem algebraischen Zahlkörper*, Math. Ann., v. 107 (1933), 731–760; [182]: *Über gewisse Ideale in einer einfachen algebra*, Act. Sci. Ind. Paris, 1934.

HASSE, H., and SCHILLING, O. [183]: *Die Normen aus einer normalen Divisionsalgebra über einem algebraischen Zahlkörper*, J. für Math., v. 174 (1936), 248–252.

HASSE, H., and SCHMIDT, F. K. [184]: *Die Struktur diskret bewerteter Körper*, J. für Math., v. 170 (1934), 4–63.

HAZLETT, O. [185]: *On the classification and invariantive characterization of nilpotent algebras*, Am. J., v. 38 (1916), 109–38; [186]: *On the theory of associative division algebras*, A. M. S. Trans., v. 18 (1917), 167–76; [187]: *On the arithmetic of a general associative algebra*, Toronto Congress Proceedings, v. 1 (1934), 185–91; [188]: *On division algebras*, A. M. S. Trans., v. 32 (1930), 912–25; [189]: *Integers as matrices*, Bologna Congress Proceedings, v. 2 (1928), 57–62.

HECKE, E. [190]: *Theorie der Algebraischen Zahlen*, Leipzig, 1923, 264 pp.

HENKE, K. [191]: *Zur arithmetischen Idealtheorie hyperkomplexer Zahlen*, Hamb. Abh., v. 11 (1935), 311–32.

HERTER, M. [192]: *Klassenzahl von Ringidealen ganzer Hurwitzscher Quaternionen und Hermitscher Formen*, Zurich dissertation, 1936, 59 pp.

HEY, K. [193]: *Analytische Zahlentheorie in Systemen hyperkomplexer Zahlen*, Hamburg dissertation, 1929.

HEYTING, A. [194]: *Die Theorie der linearen Gleichungen in einer Zahlenspezies mit nichtkommutativer Multiplikation*, Math. Ann., v. 98 (1927), 465–90.

HIRSCH, K. A. [195]: *A note on non-commutative polynomials*, London M. S. J., v. 12 (1937), 264–6.

HUA, L. K. [196]: *On a certain kind of operations connected with linear algebras*, Tôhoku J., v. 41 (1935), 222–46.

HULL, R. [197]: *Maximal orders in rational cyclic algebras of odd prime degree*, A. M. S. Trans., v. 38 (1934), 515–30; [198]: *The maximal orders of generalized quaternion division algebras*, A. M. S. Trans., v. 40 (1936), 1–11; [199]: *Note on the ideals of rational cyclic algebras of odd prime degree*, Am. J., v. 43 (1937), 384–87.

INGRAHAM, M. H. [200]: *Note on the reducibility of algebras without a finite base*, A. M. S. Bull., v. 38 (1932), 100–4; [201]: *A note on determinants*, A. M. S. Bull., v. 43 (1937), 579–80; [202]: *On a certain equation in matrices whose elements belong to a division algebra*, A. M. S. Bull., v. 44 (1938), 114–24.

INGRAHAM, M. H., and WOLF, M. C. [203]: *Relative linear sets and similarity of matrices whose elements belong to a division algebra*, A. M. S. Trans., v. 42 (1937), 16–31.

JACOBSON, N. [204]: *Non-commutative polynomials and cyclic algebras*, Ann. of Math., v. 35 (1934), 197–208; [205]: *A note on non-commutative polynomials*, ibid., 209–10; [206]: *Rational methods in the theory of Lie algebras*, Ann. of Math., v. 36 (1935), 987–91; [207]: *Totally disconnected locally compact rings*, Am. J., v. 58 (1936), 433–49; [208]: *A class of normal simple Lie algebras of characteristic zero*, Ann. of Math., v. 38 (1937), 508–14; [209]: *Pseudolinear transformations*, Ann. of Math., v. 38 (1937), 484–587; [210]: *A note on non-associative algebras*, Duke J., v. 3 (1937), 544–8; [211]: *A note on topological fields*, Am. J., v. 59 (1937), 889–894; [212]: *p-algebras of exponent p*, A. M. S. Bull., v. 43 (1937), 667–70; [213]: *Abstract derivation and Lie algebras*, A. M. S. Trans., v. 42 (1937), 206–24; [214]: *Simple Lie algebras of type A*, Ann. of Math., v. 39 (1938), 181–8.

JACOBSON, N., and TAUSSKY, O. [215]: *Locally compact rings*, P. N. A. S., v. 21 (1935), 106–8.

JORDAN, P. [216]: *Über eine Klasse nichtassoziativer hyperkomplexer Algebren*, Gött. Nach., (1932), 569–75; [217]: *Über Verallgemeinerungsmöglichkeiten der Formalismus der Quantenmechanik*, Gött. Nach., 1933, 209–14; [218]: *Über die Multiplikation quantenmechanischer Grössen*, Zeit. Phy., v. 80 (1933), 285–91.

JORDAN, P., VON NEUMANN, J., and WIGNER, E. [219]: *On an algebraic generalization of the quantum mechanical formalism*, Ann. of Math., v. 35 (1934), 29–64.

KETCHUM, P. W. [220]: *Analytic functions of hypercomplex variables*, Ann. of Math., v. 30 (1928), 641–67.

KODAIRA, K. [221]: *Über die Struktur des endlichen, vollständig primären Ringes mit verschwindendem Radikalquadrat*, Jap. J., v. 14 (1937), 15–21.

KORINEK, W. [222]: *Quadratische Körper in Quaternionenringen*, abstracted in the Zentralblatt, v. 3 (1932), 53; [223]: *Maximale kommutative Körper in einfachen Systemen hypercomplexer Zahlen*, ibid., v. 7 (1933), 293; [224]: *Une remarque concernant l'arithmétique des nombres hyperkomplexes*, loc. cit.

KÖTHE, G. [225]: *Über maximale nilpotente Unterringe und Nilringe*, Math. Ann., v. 103 (1930), 359–63; [226]: *Die Struktur der Ringe deren Restklassenring nach dem Radikal vollständig reduzible ist*, Math. Zeit., v. 32 (1930), 161–86; [227]: *Abstrakte Theorie nichtkommutativer Ringe mit einer Anwendung auf die Darstellungstheorie kontinuierlicher Gruppen*, Math. Ann., v. 103 (1930), 545–72; [228]: *Ein Beitrag zur Theorie der kommutativen Ringe ohne Endlichkeitsvoraussetzung*, Gött. Nach., (1930), 195–207; [229]: *Schiefkörper unendlichen Ranges über dem Zentrum*, Math. Ann., v. 105 (1931), 15–39; [230]: *Über Schiefkörpern mit Unterkörpern zweiter Art über dem Zentrum*, J. für Math., v. 166 (1932), 182–84; [231]: *Erweiterung des Zentrums einfacher Algebren*, Math. Ann., v. 107 (1933), 761–6.

KRULL, W. [232]: *Über verallgemeinerte endliche Abelsche Gruppen*, Math. Zeit., v. 23 (1925), 161–96.

LANDHERR, W. [233]: *Über einfache Liesche Ringe*, Hamb. Abh., v. 11 (1937), 41–64; [234]: *Liesche Ringe von Typus A*, Hans. Abh., v. 12 (1938), 200–41.

LATIMER, C. G. [235]: *Arithmetics of generalized quaternion algebras*, Am. J., v. 48 (1926), 57–66; [236]: *On the finiteness of the class number in a semi-simple algebra*, A. M. S. Bull., v. 40 (1934), 433–5; [237]: *On ideals in generalized quaternion algebras, and Hermitian forms*, A. M. S. Trans., v. 38 (1935), 436–46; [238]: *On the fundamental number of a rational generalized quaternion algebra*, Duke J., v. 1 (1935), 433–5; [239]: *On the class number of a quaternion algebra with a negative fundamental number*, A. M. S. Trans., v. 40 (1936), 318–23; [240]: *On ideals in a quaternion algebra and representation*

of integers by Hermitian forms, ibid., 439–49; [241]: *The quadratic subfields of a generalized quaternion algebra*, Duke J., v. 2 (1936), 681–4; [242]: *The classes of integral sets in a quaternion algebra*, Duke J., v. 3 (1937), 237–47.

LEFSCHETZ, S. [243]: *On the real folds of abelian varieties*, P. N. A. S., v. 5 (1919), 103–5; [244]: *Real hypersurfaces contained in abelian varieties*, ibid., 296–8; [245]: *On certain numerical invariants of algebraic varieties with application to abelian varieties*, A. M. S. Trans., v. 22 (1921), 327–482; [246]: *Selected topics on algebraic geometry*, N. R. C. Bull., v. 63 (1928), 310–95.

LEVITZKI, J. [247]: *Über vollständige reduzible Ringe und Unterringe*, Math. Zeit., v. 33 (1931), 663–91; [248]: *Über nilpotente Subringe*, Math. Ann., v. 105 (1931), 620–7; [249]: *On normal products of algebras*, Ann. of Math., v. 33 (1932), 377–402; [250]: *On automorphisms of certain rings*, Ann. of Math., v. 36 (1935), 984–92; [251]: *On the equivalence of the nilpotent elements of a semi-simple ring*, Comp. Math., v. 5 (1938), 392–402.

LITTLEWOOD, D. E. [252]: *The solution of linear congruences in quaternions*, London M. S. Pro., v. 32 (1931), 115–28; [253]: *Identical relations satisfied by an algebra*, ibid., 312–20; [254]: *On the classification of algebras*, loc. cit., v. 35 (1933), 200–40.

LITTLEWOOD, D. E., and RICHARDSON, A. R. [255]: *Fermat's equation in real quaternions*, London M. S. Pro., v. 32 (1931), 235–40; [256]: *Concomitants of polynomials in non-commutative algebra*, London M. S. J., v. 35 (1933), 325–79.

LO VOI, A. [257]: *Intorno alla construzione delle matrici di Riemann e alle loro moltiplicazioni complesse*, Cir. Mat. Pal., v. 55 (1931), 287–357; [258]: Ibid., 477–88.

MAASS, H. [259]: *Beweis des Normensatzes in einfachen hyperkomplexen Systemen*, Hans. Abh., v. 12 (1937), 64–9.

MACDUFFEE, C. C. [260]: *On the independence of the first and second matrices of an algebra*, A. M. S. Bull., v. 35 (1929), 344–9; [261]: *An introduction to the theory of ideals in linear associative algebras*, A. M. S. Trans., v. 31 (1929), 71–90; [262]: *The discriminant matrices of a linear associative algebra*, Ann. of Math., v. 32 (1931), 60–6; [263]: *The discriminant matrix of a semi-simple algebra*, A. M. S. Trans., v. 33 (1931), 425–32; [264]: *Ideals in linear algebras*, A. M. S. Bull., v. 37 (1931), 841–53; [265]: *The Theory of Matrices*, Erg. der Math., v. 2, 1933, 110 pp.; [266]: *Matrices with elements in a principal ideal ring*, A. M. S. Bull., v. 39 (1933), 564–84.

MAEDA, F. [267]: *Ring-decomposition without chain condition*, Hiro. J., v. 8 (1938), 145–67.

MALCEV, A. [268]: *On the immersion of an algebraic ring into a field*, Math. Ann., v. 113 (1937), 686–91.

MCCOY, N. H. [269]: *On the characteristic roots of matrix polynomials*, A. M. S. Bull., v. 42 (1936), 592–600.

MCCOY, N. H. and MONTGOMERY, D. [270]: *A representation of generalized Boolean rings*, Duke J., v. 3 (1937), 455–9.

MOISIL, G. C. [271]: *Remarques sur quelques types d'algèbres non-associatives*, Jassy Ann., v. 20 (1935), 10–36.

MOLIEN, T. [272]: *Zahlensysteme mit einer Haupteinheit*, abstracted in Zentralblatt, v. 17 (1937–8), 53.

MORI, S. [273]: *Über allgemeine Multiplikationsringe*, I, Hiro. J., v. 4 (1934), 1–26; [274]: Ibid., II, 99–109; [275]: *Über primäre Ringe*, loc. cit., v. 5 (1935), 131–9; [276]: *Über Totalnullteiler kommutativer Ringe mit abgeschwächtem U-satz*, I, loc. cit., v. 6 (1936), 139–46; [277]: Ibid., II, 257–69.

MORIYA, M. [278]: *Divisionsalgebren über einem $\mathfrak{p}$-adischen Zahlkörper eines unendlichen Zahlkörpers*, Imp. Aca. Tok. Pro., v. 12 (1936), 183–4.

MORIYA, M., and SCHILLING, O. F. G. [279]: *Zur Klassenkörpertheorie über unendlichen perfekten Körpern*, Hokk. J., v. 5 (1937), 189–205.

MOUFGANG, R. [280]: *Alternativkörper und der Satz vom vollständigen Vierseit (D_9)*, Hamb. Abh., v. 9 (1933), 207–22; [281]: *Zur Struktur Alternativkörper*, Math. Ann., v. 110 (1934), 416–30; [282]: *Einige Untersuchungen über geordnete Schiefkörper*, J. für Math., v. 176 (1937), 203–23.

MURRAY, F. J., and VON NEUMANN, J. [283]: *On rings of operators*, Ann. of Math., v. 37 (1936), 116–229; [284]: Ibid., II, A. M. S. Trans., v. 41 (1937), 208–48.

NAGUMO, M. [285]: *Einige analytische Untersuchungen in linearen metrischer Ringen*, Jap. J., v. 13 (1936), 61–80.

NAKAMURA, T. [286]: *Ein Satz über p-adische Schiefkörper*, Imp. Aca. Tok. Pro., v. 10 (1934), 198–9.

NAKAYAMA, T. [287]: *Über die Definition der Shodaschen Discriminante eines normalen einfachen hyperkomplexen Systeme*, Imp. Aca. Tok. Pro., v. 10 (1934), 447–9; [288]: *Über das direkte Produkt zweier einfachen Algebren mit zueinander teilerfremden p-Indizen*, Jap. J., v. 12 (1935), 27–36; [289]: *Über das direkte Zerlegung einer Divisionsalgebra*, Jap. J., v. 12 (1935), 65–70; [290]: *Über die Beziehungen zwischen den Faktorensystemen und der Normklassengruppe eines galoisschen Erweiterungskörpers*, Math. Ann., v. 112 (1935), 85–91; [291]: *Eine Bemerkung über die Summe und den Durchschmitt von zwei Idealen in einer Algebra*, Imp. Aca. Tok. Pro., v. 12 (1936), 179–82; [292]: *Maximalordnungen und Erweiterung des Koeffizienten-Körpers eines hyperkomplexen Systems*, Jap. J., v. 13 (1937), 333–59; [293]: *Über die Algebren über einem Körper von der Primzahlcharakteristik*, Imp. Aca. Tok. Pro., v. 11 (1935), 305–6; [294]: Ibid., II, v. 12 (1936), 113–4; [295]: *Über die Klassifikation halblinearer Transformationen*, P.-M. S. Jap., v. 19 (1937), 99–107; [296]: *Divisionsalgebren über diskret bewerteten perfekten Körpern*, J. für Math., v. 178 (1937), 11–3; [297]: *Some studies on regular representations, induced representations, and modular representations*, Ann. of Math., v. 39 (1938), 361–9.

NAKAYAMA, T., and SHODA, K. [298]: *Über die Darstellung einer endlichen Gruppe durch halblinearen Transformationen*, Jap. J., v. 12 (1936), 109–22.

NEHRKORN, H. [299]: *Über absolute Idealklassengruppen und Einheiten in algebraischen Zahlkörpern*, Hamb. Abh., v. 9 (1933), 318–34.

VON NEUMANN, J. [300]: *On regular rings*, P. N. A. S., v. 22 (1936), 707–13; [301]: *Algebraic theory of continuous geometries*, P. N. A. S., v. 23 (1937), 16–22; [302]: *Continuous rings and their arithmetics*, ibid., v. 23 (1937), 341–9.

VON NEUMANN, J., and STONE, M. H. [303]: *The determination of representative elements in the residual classes of a Boolean algebra*, Fund. Math., v. 25 (1935), 353–78.

NOETHER, E. [304]: *Der Diskriminantensatz für die Ordnungen eines algebraischen Zahl oder Funktionenkörpers*, J. für Math., v. 157 (1927), 82–104; [305]: *Hyperkomplexe Grössen und Darstellungstheorie*, Math. Zeit., v. 30 (1929), 641–92; [306]: *Hypercomplexe Systeme in ihre Beziehungen zur kommutativen Algebra und Zahlentheorie*, Zurich Congress Proceedings (1932), 189–94; [307]: *Nichtkommutative Algebra*, Math. Zeit., v. 37 (1933), 514–41; [308]: *Der Hauptgeschlechtssatz für relative-galoissche Zahlkörper*, Math. Ann., v. 108 (1933), 411–19; [309]: *Zerfallende verschränkte Produkte und ihre Maximalordnungen*, Act. Sci. Ind. Paris, 1934.

NOWLAN, F. S. [310]: *On the direct product of a division algebra and a total matric algebra*, A. M. S. Bull., v. 36 (1930), 265–8.

NOWLAN, F. S., and HULL, R. [311]: *Sets of integral elements of certain rational division algebras*, R. S. Can. Trans., v. 31 (1937), 163–84.

OLSON, H. L. [312]: *Linear congruences in a general arithmetic*, Ann. of Math., v. 28 (1926), 237–9; [313]: *Doubly divisible quaternions*, Ann. of Math., v. 31 (1930), 371–4.

ORE, O. [314]: *Linear equations in non-commutative fields*, Ann. of Math., v. 32 (1931), 463–77; [315]: *Theory of noncommutative polynomials*, Ann. of Math., v. 34 (1933), 480–508; [316]: *On a special class of polynomials*, A. M. S. Trans., v. 35 (1933), 559–84.

OSIMA, M. [317]: *Über die Darstellung einer Gruppe durch halblineare Transformationen*, P.-M. S. Jap., v. 20 (1938), 1–5.

PALL, G. [318]: *The quaternionic congruence* $tat \equiv b \pmod{g}$, *and the equation* $h(gn + 1) = x_1^2 + x_2^2 + x^2$, Am. J., v. 59 (1937), 895–913.

PERLIS, S. [319]: *Maximal orders in rational cyclic algebras of composite degree*, A. M. S. Trans., v. 46 (1939), 82–96.

RANUM, A. [320]: *The groups belonging to a linear associative algebra*, Am. J., v. 49 (1927), 285-308.

RAUTER, H. [321]: *Quaternionen mit Komponenten aus einem Körper von Primzahlcharakteristik*, Math. Zeit., v. 29 (1929), 234-63.

REES, M. S. [322]: *Division algebras associated with an equation whose group has four generators*, Am. J., v. 54 (1932), 51-65.

REICHARDT, H. [323]: *Die Diskriminate einer normalen einfachen Algebra*, J. für Math., v. 173 (1935), 31-4.

RICHARDSON, A. R. [324]: *Hypercomplex determinants*, Mess. Math., v. 55 (1926), 145-52; [325]: *Concomitants relating to a division algebra*, ibid., 175-82; [326]: *Equations over a division algebra*, loc. cit., v. 57 (1928), 1-6; [327]: *Simultaneous linear equations over a division algebra*, London M. S. Pro., v. 28 (1928), 395-420.

RINEHART, R. F. [328]: *Some properties of the discriminant matrices of a linear associative algebra*, A. M. S. Bull., v. 42 (1936), 570-6.

RINGLEB, F. [329]: *Beiträge zur Funktionentheorie in hyperkomplexen Systemen* I, Cir. Mat. Pal., v. 57 (1933), 311-40; [330]: *Bemerkung zur Arbeit des Verfassers*: "*Beiträge zur Funktionentheorie in hyperkomplexen Systemen* I," ibid., 476-7.

ROSATI, C. [331]: *Sulle matrici di Riemann*, Cir. Mat. Pal., v. 53 (1929), 79-134.

ROSENBLUTH, E. [332]: *Die arithmetische Theorie und die Konstruktion der Quaternionenkörper auf klassenkörpertheoretische Grundlage*, Monat., v. 41 (1934), 85-125.

RUSAM, F. [333]: *Matrizenringe mit Koeffizienten aus endlichen Ringen ganzer Zahlen*, Erlangen dissertation, 1931, 71 pp.

SCHILLING, O. F. G. [334]: *Über gewisse Beziehungen zwischen der Arithmetik hyperkomplexer Zahlsysteme und algebraischer Zahlkörper*, Math. Ann., v. 111 (1935), 372-98; [335]: *Über die Darstellungen endlicher Gruppen*, J. für Math., v. 174 (1936), 188; [336]: *Zur algebraischen Theorie der Funktionenkörper mehrerer Variablen*, J. für Math., v. 175 (1936), 1-5; [337]: *Einheitentheorie in rationalen hyperkomplexen Systemen*, J. für Math., v. 175 (1936), 246-51; [338]: *Arithmetic in a special class of algebras*, Ann. of Math., v. 38 (1937), 116-19; [339]: *Some remarks on class field theory over infinite fields of algebraic numbers*, Am. J., v. 59 (1937), 414-22; [340]: *Class fields of infinite degree over p-adic number fields*, Ann. of Math., v. 38 (1937), 469-76; [341]: *The structure of certain rational infinite algebras*, Duke J., v. 3 (1937), 303-10; [342]: *Arithmetic in fields of formal power series in several variables*, Ann. of Math., v. 38 (1937), 551-76; [343]: *The structure of local class field theory*, Am. J., v. 40 (1938), 75-100; [344]: *A generalization of local class field theory*, ibid., 667-704.

SCHILLING, O. F. G., and HASSE, H. [345]: *Die Normen aus einer normalen Divisionsalgebra über einem algebraischen Zahlkörper*, J. für Math., v. 174 (1936), 248-52.

SCHILLING, O. F. G., and MORIYA, M. [346]: *Zur Klassenkörpertheorie über unendlichen perfekten Körpern*, Hokk. J., v. 5 (1937), 189-205; [347]: *Divisionsalgebren über unendlichen perfekten Körpern*, Hokk. J., v. 6 (1937), 103-11.

SCHMIDT, F. K. [348]: *Zur Klassenkörpertheorie im Kleinem*, J. für Math., v. 162 (1930), 155-68.

SCHOENEBERG, B. [349]: *Indefinite Quaternionen und Modulfunktionen*, Math. Zeit., v. 40 (1936), 94-109.

SCORZA, G. [350]: *Corpi Numerici e Algebra*, Messina, 1921; [351]: *Intorno alla teoria generale delle matrici di Riemann*, Cir. Mat. Pal., v. 41 (1916), 263-380; [352]: *Le algebre di ordine qualunque e le matrici di Riemann*, Cir. Mat. Pal., v. 45 (1921), 1-204; [353]: *Sopra un teorema fondamentale della teoria delle algebre*, Atti Lin., v. 20 (1934), 65-72; [354]: *Sulla struttura delle algebre pseudonulle*, ibid., 143-9; [355]: *Sulle algebre pseudonulle di ordine massimo*, Ann. di Mat., v. 14 (1935), 1-14; [356]: *Sopra una classe di algebre pseudonulle*, Atti Tor., v. 70 (1935), 196-211; [357]: *A propositio di un recente lavoro di A. A. Albert*, Atti Lin., v. 21 (1935), 727-32; [358]: *Le algebre del* 3.° *ordine*, Atti Nap., v. 20 (1938), no. 13, 1-14; [359]: *Le algebre del* 4.° *ordine*, Atti Nap., v. 20

(1935), no. 14, 1–83; [360]: *Nuovi contributi alla teoria generale delle algebre*, Atti Lin., v. 23 (1936), 915–20; [361]: *Nuovi contributi alla teoria generale delle algebra*, Rome Rend., v. 1 (1936), 59–82.

SHAW, J. B. [362]: *Synopsis of Linear Associative Algebras*, Washington, 1907.

SHODA, K. [363]: *Über die Automorphismen einer endlichen Abelschen Gruppe*, Math. Ann., v. 100 (1928), 674–86; [364]: *Über die Einheitengruppe eines endlichen Ringes*, I, Math. Ann., v. 102 (1929), 674–86; [365]: Ibid., II, Imp. Aca. Tok. Pro., v. 6 (1930), 93–6; [366]: *Über die mit einer Matrix vertauschbaren Matrizen*, Math. Zeit., v. 29 (1929), 696–712; [367]: *Über die Galoissche Theorie der halbeinfachen hyperkomplexen Systeme*, Math. Ann., v. 107 (1932), 252–8; [368]: *Bemerkungen über die Faktorensysteme einfacher hyperkomplexer Systeme*, Imp. Aca. Tok. Pro., v. 10 (1933), 57–70; [369]: *Über die endlichen Gruppen der Algebrenklassen mit einem Zerfällungskörper*, Imp. Aca. Tok. Pro., v. 10 (1934), 21–30; [370]: *Ein Kriterium für normale einfache hyperkomplexe Systeme*, Imp. Aca. Tok. Pro., v. 10 (1934), 195–7; [371]: *Diskriminantensatz für normale einfache hyperkomplexe Systeme*, Imp. Aca. Tok. Pro., v. 10 (1934), 315–7; [372]: *Diskriminantenformel für normale einfache hyperkomplexe Systeme*, Imp. Aca. Tok. Pro., v. 10 (1934), 318–21; [373]: *Hyperkomplexe Bedeutung der invarianten Idealklassen relativ-galoisscher Zahlkörper*, Imp. Aca. Tok. Pro., v. 12 (1935), 59–64; [374]: *Über die Maximalordnungen einer normalen einfachen Algebra mit einem unverzweiglen galoisschen Zerfällungskörper*, Jap. J., v. 13 (1937), 367–73.

SHODA, K., and NAKAYAMA, T. [375]: *Über das Produkt zweier Algebrenklassen mit zueinander primen Diskriminanten*, Imp. Aca. Tok. Pro., v. 10 (1934), 443–6.

SHOVER, G. [376]: *Class number in a linear associative algebra*, A. M. S. Bull., v. 39 (1933), 610–34.

SHOVER, G., and MACDUFFEE, C. C. [377]: *Ideal multiplication in a linear algebra*, A. M. S. Bull., v. 37 (1931), 434–8.

SKOLEM, T. [378]: *Zur Theorie der associativen Zahlensysteme*, Oslo, 1937.

SMITH, G. W. [379]: *Nilpotent algebras generated by two units i and j, such that i^2 is not an independent unit*, Am. J., v. 41 (1919), 143–64.

SPAMPINATO, N. [380]: *Sulle funzioni di una variabile, in un'algebra complessa, ad n unità, dotata di modulo*, Cir. mat. Pal., v. 57 (1933), 235–72; [381]: Ibid., II, v. 58 (1934), 105–43; [382]: Loc. cit., III, v. 59 (1935), 185–227; [383]: *Sulle funzioni totalmente derivabili in un'algebra reale e complessa dotato di modulo*, I, Atti Lin., v. 21 (1935), 621–5; [384]: Ibid., II, 683–7; [385]: *Una proprietà caratteristica delle funzioni totalmente derivabili*, I, Atti Lin., v. 21 (1935), 625–33; [386]: Ibid., II, 688–92.

SPEISER, A. [387]: *Gruppendeterminante und Körperdiskriminante*, Math. Ann., v. 77 (1916), 546–62; [388]: *Allgemeine Zahlentheorie*, Nat. Ges. Zur. V., v. 71 (1926), 8–48; [389]: *Idealtheorie in rationalen algebren*, Dickson's *Algebren*, Chapter 13; [390]: *Zahlentheorie in rationalen Algebren*, Comm. Math. Helv., v. 8 (1936), 391–406.

STAUFFER, R. [391]: *The construction of a normal basis in a separable normal extension field*, Am. J., v. 58 (1936), 585–97.

STONE, M. H. [392]: *The theory of representations for Boolean algebras*, A. M. S. Trans., v. 39 (1936), 37–111.

TANNAKA, T. [393]: *Zyklische Zerfällungskörper der einfachen Ringe über dem algebraischen Funktionenkörper*, Toh. Imp. U., Sci. Rep., v. 24 (1935), 165–73.

TAUSSKY, O. [394]: *Analytical methods in hypercomplex systems*, Comp. Math., v. 3 (1936), 399–407; [395]: *Rings with noncommutative addition*, Calc. M. S. Bull., v. 28 (1936), 245–6.

TEICHMÜLLER, O. [396]: *Verschränkte Produkte mit Normalringen*, Deut. Math., 1936, 92–102; [397]: *Multiplikation zyklischer Normalringe*, ibid., 197–238; [398]: *p-Algebren*, loc. cit., 362–88; [399]: *Diskret bewertete perfekte Körper mit unvollkommenem Restklassenkörper*, J. für Math., v. 176 (1937), 141–52; [400]: *Zerfällende zyklische p-algebren*, ibid., 157–60.

TELLER, J. H. D. [401]: *A class of quaternion algebras*, Duke J., v. 2 (1936), 280–6.

TOYODA, K. [402]: *On Casimir's theorem on semi-simple continuous groups*, Jap. J., v. 12 (1935), 17–20; [403]: *On the structure of Lie's continuous groups*, Toh. Imp. U. Sci. Rep., v. 25 (1936), 338–43.

TROST, E. [404]: *Über die Struktur der normalen Divisionsalgebren*, Zurich dissertation, 1933, 33 pp.

TSEN, C. C. [405]: *Divisionalgreben über Funktionenkörpern*, Gött. Nach. (1933), 335–9; [406]: *Algebren über Funktionenkörpern*, Göttingen dissertation, 1934; [407]: *Zur Stufentheorie der quasialgebraisch-Abgeschlossenheit kommutativer Körper*, Chi. M. S. J., v. 1 (1936), 81–92.

VANDIVER, H. S. [408]: *Note on an associative distributive algebra in which the commutative law of addition does not hold*, A. M. S. Bull., v. 42 (1936), 857–9.

VENKATARAYUDU, T. [409]: *On the linear algebra of classes of elements in a finite abelian group*, Ind. Aca. Pro., v. 5 (1937), 433–6; [410]: *Ideal theory of the abelian group algebra*, Ind. Aca. Pro., v. 7 (1938), 118–29.

VENKOV, B. [411]: *Über der Arithmetik der Quaternionen*, Bull. Aca. Sci. U. S. S. R., v. 16 (1922), 205–46; [412]: Ibid. (1929).

VAN DER WAERDEN, B. L. [413]: *Moderne Algebra*, vol. 1, Berlin, 1930; [414]: Ibid., vol. 2; [415]: *Gruppen von linearen Transformationen*, Erg. der Math., v. 4 (1935); [416]: *Die Klassifikation der einfachen Lieschen Gruppen*, Math. Zeit., v. 37 (1933), 446–62.

WAJNSTEJN, D. [417]: *Über die Clifford-Lipschitzen hyperkomplexen Zahlensysteme*, Soc. Pol. Mat. Ann., v. 16 (1938), 65–83; [418]: *α-Matrizen und Clifford-Zahlen*, Soc. Pol. Mat. Ann., v. 16 (1938), 162–75.

WARNING, E. [419]: *Bemerkung zur vorstehenden Arbeit von Herrn Chevalley*, Hamb. Abh., v. 11 (1935), 76–83.

WEBER, W. [420]: *Verschränkte Produkte mit Normalringen*, Deut. Math., v. 1 (1936), 90–2.

WEDDERBURN, J. H. M. [421]: *On a theorem in hypercomplex numbers*, R. S. Edin. Pro., v. 26 (1905), 1–3; [422]: *Note on hypercomplex numbers*, Edin. M. S. Pro., v. 25 (1906), 1–3; [423]: *On certain theorems in determinants*, Edin. M. S. Pro., v. 27 (1908), 1–3; [424]: *A theorem on finite algebras*, A. M. S. Trans., v. 6 (1905), 349–52; [425]: *On hypercomplex numbers*, London M. S. Pro., v. 6 (1907), 77–118; [426]: *A type of primitive algebra*, A. M. S. Trans., v. 15 (1914), 162–6; [427]: *On division algebras*, A. M. S. Trans., v. 22 (1921), 129–35; [428]: *Algebras which do not possess a finite basis*, A. M. S. Trans., v. 26 (1924), 395–426; [429]: *A theorem on simple algebras*, A. M. S. Bull., v. 31 (1925), 11–13; [430]: *Non-commutative domains of integrity*, J. für Math., v. 167 (1931), 129–41; [431]: *Boolean linear associative algebra*, Ann. of Math., v. 35 (1934), 185–94; [432]: *Lectures on Matrices*, A. M. S. Coll., v. 17, 1934, 200 pp.; [433]: *Note on algebras*, Ann. of Math., v. 38 (1937), 854–6; [434]: *A special linear associative algebra*, Edin. M. S. Pro., v. 5 (1938), 169–70.

WEYL, H. [435]: *Theorie der Darstellung kontinuierlicher halbeinfacher Gruppen*, Math. Zeit., v. 23 (1925), 211–309; [436]: Ibid., II, v. 24 (1926), 328–76; [437]: Loc. cit., III, 377–95; [438]: Loc. cit., *Appendix*, 789–91; [439]: *Idee der Riemannschen Fläche*, 2d edition, Leipzig, 1923; [440]: *On generalized Riemann matrices*, Ann. of Math., v. 35 (1934), 714–29; [441]: *Generalized Riemann matrices and factor sets*, Ann. of Math., v. 37 (1936), 709–45; [442]: *Note on matric algebras*, Ann. of Math., v. 38 (1937), 477–83; [443]: *Commutator algebra of a finite group of collineations*, Duke J., v. 3 (1937), 200–12; [444]: *The Classical Groups, Their Invariants, and Representations*, Princeton, 1939, 312 pp.

WICHMANN, W. [445]: *Anwendungen der p-adischen Theorie im Nichtkommutativen*, Monat., v. 44 (1936), 203–24.

WILLIAMSON, J. [446]: *The idempotent and nilpotent elements of a matrix*, Am. J., v. 59 (1936), 747–58.

WILSON, R. [447]: *The equation $px = xq$ in a linear associative algebra*, London M. S. Pro., v. 30 (1930), 357–66.

WITT, E. [448]: *Über die Kommutativität endlicher Schiefkörper*, Gött. Nach., 1931, 413; [449]: *Gegenbeispiel zum Normensatz*, Math. Zeit., v. 39 (1934), 462–7; [450]: *Riemann-Rochscher Satz und ζ-Funktion im Hyperkomplexen*, Math. Ann., v. 110 (1934), 12–28; [451]: *Zerlegung reeler algebraischer Funktionen im Quadrate; Schiefkörper in reellen Funktionen*, J. für Math., v. 171 (1934), 4–11; [452]: *Zwei Regeln über vershränkte Produkte*, J. für Math., v. 173 (1935), 191–2; [453]: *Konstruktion von galoisschen Körpern der Charakteristik p zu vorgegebener Gruppe der Ordnung p^l*, J. für Math., v. 174 (1936), 237–45; [454]: *Theorie der quadratischen Formen in beliebigen Körpern*, J. für Math., v. 176 (1937), 31–44; [455]: *Zyklische Körper und Algebren der Charakteristik p vom Grad, p^n*, ibid., 126–40; [456]: *Schiefkörper über diskret bewerteten Körpern*, ibid., 153–6; [457]: *Treue Darstellung Liescher Ringe*, J. für Math., v. 177 (1937), 152–60.

WOLF, L. A. [458]: *Similarity of matrices in which the elements are real quaternions*, A. M. S. Bull., v. 42 (1936), 737–43.

WYANT, E. K. [459]: *The ideals in the algebra of generalized quaternions over the rational number field*, abstract in A. M. S. Bull., v. 36 (1930), 204.

YOSIDA, K. [460]: *On the group imbedded in the metrical complete ring*, Jap. J., v. 13 (1936), 7–26; [461]: *A theorem concerning the semi-simple Lie group*, Tôhoku J., v. 44 (1937), 81–4.

YOUNG, J. W. [462]: *A new formulation for general algebra*, Ann. of Math., v. 29 (1927), 47–60.

ZASSENHAUS, H. [463]: *Lehrbuch der Gruppentheorie*, Leipzig, 1937.

ZORN, M. [464]: *Theorie der alternativen Ringe*, Hamb. Abh., v. 8 (1930), 123–47; [465]: *Note zur analytischen hyperkomplexen Zahlentheorie*, Hamb. Abh., v. 9 (1933), 197–201; [466]: *Alternativkörper und quadratische systeme*, ibid., 395–402; [467]: *The automorphisms of Cayley's non-associative algebra*, P. N. A. S., v. 21 (1935), 355–8; [468]: *On a theorem of Engel*, A. M. S. Bull., v. 43 (1937), 401–4.

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