

American Mathematical Society

Colloquium Publications

Volume 58

Eisenstein Series and Automorphic L -Functions

Freydoon Shahidi



Eisenstein Series and Automorphic L -Functions

American Mathematical Society

Colloquium Publications

Volume 58

Eisenstein Series and Automorphic L -Functions

Freydoon Shahidi



American Mathematical Society
Providence, Rhode Island

EDITORIAL COMMITTEE

Paul J. Sally, Jr., Chair
Yuri Manin
Peter Sarnak

2000 *Mathematics Subject Classification*. Primary 11F70, 11R42;
Secondary 22E50, 22E55.

The tables of Dynkin diagrams in the appendices originally appeared on pp. 558–568 in: F. Shahidi, *On the Ramanujan conjecture and finiteness of poles for certain L -functions*, *Annals of Math.* **127** (1988), 547–584. Reprinted with permission.

For additional information and updates on this book, visit
www.ams.org/bookpages/coll-58

Library of Congress Cataloging-in-Publication Data

Shahidi, Freydoon.

Eisenstein series and automorphic L -functions / Freydoon Shahidi.

p. cm. – (Colloquium publications ; v. 58)

Includes bibliographical references and index.

ISBN 978-0-8218-4989-7 (alk. paper)

1. Eisenstein series. 2. L -functions. 3. Automorphic functions. I. Title.

QA353.A9S53 2010

515'.9—dc22

2010010759

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy a chapter for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294 USA. Requests can also be made by e-mail to reprint-permission@ams.org.

© 2010 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.
Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 15 14 13 12 11 10

Contents

Introduction	1
Chapter 1. Reductive Groups	7
§1.1. Preliminaries on reductive groups	7
§1.2. Parabolic subgroups	8
§1.3. Adelization	11
§1.4. L -groups	12
§1.5. Parabolic subgroups of L -groups	14
§1.6. Appendices	15
Chapter 2. Satake Isomorphisms	23
§2.1. Unramified characters and representations	23
§2.2. Satake isomorphism	27
§2.3. Connection with L -groups and parameters; Local unramified L -functions	29
§2.4. Connection with global theory	32
§2.5. Appendix: Absolute convergence of L -functions	34
Chapter 3. Generic Representations	45
§3.1. Generic characters and representations	45
§3.2. Globally generic representations and Whittaker models	46
§3.3. Induced representations and functionals	49
§3.4. Holomorphy of p -adic Whittaker functionals	52
§3.5. Unramified Whittaker functions	60
§3.6. The Archimedean case	73
§3.7. Appendix: Remarks on representation theory	79
Chapter 4. Intertwining Operators	81
§4.1. Definition and basic properties	81
§4.2. Factorization of intertwining operators	84
§4.3. The unramified case	89
§4.4. Restriction of scalars and L -functions	95
Chapter 5. Local Coefficients	99
§5.1. Definition and basic properties	99
§5.2. Further properties	103
§5.3. Plancherel measures	105
Chapter 6. Eisenstein Series	107
§6.1. Preliminaries and main facts	107
§6.2. Computation of the constant term	111

§6.3. Connection with L -functions	115
Chapter 7. Fourier Coefficients of Eisenstein Series	121
§7.1. The non-constant term	121
§7.2. Non-vanishing of L -functions	127
Chapter 8. Functional Equations	129
§8.1. Crude functional equations	129
§8.2. Explicit formulas for local coefficients	131
§8.3. Multiplicativity and the main theorem on γ -functions	140
§8.4. Definition of L -functions and root numbers	144
§8.5. Connection with reducibility and complementary series	149
Chapter 9. Further Properties of L -functions	153
§9.1. Finiteness of poles	153
§9.2. Boundedness in vertical strips	156
§9.3. Holomorphy of L -functions under highly ramified twists	163
Chapter 10. Applications to Functoriality	167
§10.1. Rankin-Selberg L -functions	167
§10.2. A converse theorem	169
§10.3. Functorial transfer from $GL(2) \times GL(3)$ to $GL(6)$	169
§10.4. Third symmetric powers for $GL(2)$	172
§10.5. Classical groups	175
§10.6. Ramanujan-Selberg estimates for Maass forms	177
§10.7. Other aspects and directions of the method	178
Appendices: Tables of Dynkin Diagrams	181
Appendix A. Simple factors (split cases)	183
Appendix B. Non-simple factors (split cases)	187
Appendix C. Non-split quasisplit cases	193
Bibliography	199
Index	207

Bibliography

- [A1] J. Arthur, *Eisenstein series and trace formula*, in Automorphic Forms, Representations and L -functions, in Proc. Sympos. Pure Math. 33, Part 1, 1979, pp. 253–274.
- [A2] J. Arthur, *Intertwining operators and residues I. Weighted characters*, J. Funct. Anal. **84** (1989), 19–84.
- [A3] J. Arthur, *On some problems suggested by the trace formula*, Lie Group Representations II, Proceedings, Univ. of Maryland (B. Herb, S. Kudla, R. Lipsman and J. Rosenberg, ed.), Springer LN, vol. 1041, 1982-83, pp. 1–49.
- [A4] J. Arthur, *Unipotent automorphic representations: Conjectures*, Astérisque **171–172** (1989), 13–71.
- [A5] J. Arthur, *Unipotent automorphic representations: Global motivation*, Automorphic Forms, Shimura Varieties and L -functions (L. Clozel and S.J. Milne, ed.), vol. 1, Perspect. Math. **10**, Academic Press, Boston, 1990, pp. 1–75.
- [A6] J. Arthur, *Endoscopic L -functions and a combinatorial identity*, Dedicated to H.S.M. Coxeter, Canad. J. Math. **51** (1999), 1135–1148.
- [A7] J. Arthur, *An introduction to the trace formula*, Harmonic Analysis, the Trace Formula, and Shimura Varieties, Clay Math. Proc. **4**, Amer. Math. Soc., Providence, RI, 2005, pp. 1–263.
- [A8] J. Arthur, *The principle of functoriality*, Bull. Amer. Math. Soc. (N.S.) **40:1** (2002), 39–53; Mathematical Challenges of the 21st century (Los Angeles, CA) (2000).
- [A9] J. Arthur, *The Endoscopic Classification of Representations: Orthogonal and Symplectic Groups*, Colloquium Publication Series, AMS (to appear).
- [AC] J. Arthur and L. Clozel, *Simple algebras, Base change, and the Advanced Theory of the Trace Formula*, Annals of Math. Studies **120**, Princeton University Press, 1989.
- [As] M. Asgari, *Local L -functions for split spinor groups*, Canad. J. Math. **154** (2002), 673–693.
- [ASh1] M. Asgari and F. Shahidi, *Generic transfer for general spin groups*, Duke Math. J. **132** (2006), 137–190.
- [ASh2] M. Asgari and F. Shahidi, *Generic transfer from $GSp(4)$ to $GL(4)$* , Compositio Mathematica **142** (2006), 541–550.
- [Au] A.-M. Aubert, *Dualité dans le groupe de Grothendieck de la catégorie des représentations lisses de longueur finie d'un groupe réductif p -adique*, Transactions of the American Mathematical Society **347** (1995), 2179–2189; and Erratum, Transactions of the American Mathematical Society **348** (1996), 4687–4690.
- [Ba] D. Ban, *The Aubert involution and R -groups*, Ann. Scient. Éc. Norm. Sup. **35** (2002), 673–693.
- [BaJa] D. Ban and D. Jantzen, *Duality and the Normalization of Standard Intertwining Operators*, Manuscripta Mathematica **115**, no. 4 (2004), 401–415.
- [Bl] D. Blasius, *Hilbert modular forms and the Ramanujan conjecture*, Noncommutative geometry and number theory, 35–56, Aspects Math., E37, Vieweg, Wiesbaden, 2006.
- [B1] A. Borel, *Automorphic L -functions*, in Automorphic Forms and Automorphic Representations, Proc. Sympos. Pure Math. **33; II**, Amer. Math. Soc., Providence, RI, 1979, pp. 27–61.
- [B2] A. Borel, *Linear Algebraic Groups*, GTM126, Springer-Verlag, New York, 1997.
- [BJ] A. Borel and H. Jacquet, *Automorphic forms and automorphic representations*, in Automorphic Forms and Automorphic Representations, Proc. Sympos. Pure Math. 33, I, AMS, Providence, RI, 1979, pp. 189–202.

- [BWa] A. Borel and N. Wallach, *Continuous Cohomology, Discrete Subgroups, and Representations of Reductive Groups*, Annals of Math. Studies **94**, Princeton Univ. Press (1980).
- [Bou] N. Bourbaki, *Groupes et Algèbres de Lie*, Chap. 4–6, Paris, Hermann, 1968.
- [BBF] B. Brubaker, D. Bump and S. Friedberg, *Weyl group multiple Dirichlet series, Eisenstein series, and crystal basis*, Annals of Math.; in press.
- [BrTi] F. Bruhat and J. Tits, *Groupes réductifs sur un corps local II*, Publ. Math. IHES **60** (1984), 197–376.
- [BuHe] C.J. Bushnell and G. Henniart, *On certain dyadic representations (Appendix to [KSh2])*, Ann. of Math. **155** (2002), 883–893.
- [BuKu] C. Bushnell and P.C. Kutzko, *The admissible dual of $GL(N)$ via compact open subgroups*, Annals of Mathematics Studies, 129, Princeton University Press, Princeton, NJ, 1993.
- [Car] P. Cartier, *Representations of p -adic Groups: A Survey*, in Automorphic Forms and Automorphic Representations, Proc. Sympos., Pure Math. **33; I**, Amer. Math. Soc., Providence, RI, 1979, pp. 111–155.
- [Ca1] W. Casselman, *Introduction to the theory of admissible representations of p -adic reductive groups*, preprint.
- [Ca2] W. Casselman, *The unramified principal series of p -adic groups I. The spherical function*, Comp. Math. **40** (1980), 387–406.
- [Ca3] W. Casselman, *Canonical extensions of Harish–Chandra modules*, Canad. J. Math. **41** (1989), 315–438.
- [CM] W. Casselman and D. Miličič, *Asymptotic behavior of matrix coefficients of admissible representations*, Duke Math. J. **49** (1982, no. 4), 869–930.
- [CSH] W. Casselman and F. Shahidi, *On irreducibility of standard modules for generic representations*, Ann. Scient. Éc. Norm. Sup. **31** (1998), 561–589.
- [CS] W. Casselman and J.A. Shalika, *The unramified principal series of p -adic groups II; The Whittaker function*, Comp. Math. **41** (1980), 207–231.
- [CHT] L. Clozel, M. Harris and R. Taylor, *Automorphy for some l -adic lifts of automorphic mod l Galois representations*, Publ. Math. IHES **108** (2008), 1–181.
- [CLL] L. Clozel, J-P. Labesse and R.P. Langlands, *Morning Seminar on the Trace Formula, Lecture Notes*, Institute for Advanced Study, Princeton, NJ, 1984.
- [C1] J.W. Cogdell, *L -functions and converse theorems for GL_n* , Automorphic Forms and Applications, IAS/Park City Math. Ser. **12**, Amer. Math. Soc., Providence, RI, 2007, pp. 97–177.
- [C2] J.W. Cogdell, *Lectures on L -functions, Converse Theorems, and Functoriality for $GL(n)$* , in Lectures on Automorphic L -functions, Fields Institute Monographs, AMS, 2004, pp. 3–96.
- [CPS1] J.W. Cogdell and I.I. Piatetski–Shapiro, *Converse theorems for GL_n* , Publ. Math. IHES **79** (1994), 157–214.
- [CPS2] J.W. Cogdell and I.I. Piatetski–Shapiro, *Converse theorems for GL_n II*, J. Reine Angew. Math. **507** (1999), 165–188.
- [CPS3] J.W. Cogdell and I.I. Piatetski–Shapiro, *Converse Theorems, Functoriality and Applications to Number Theory*, Proceedings of the International Congress of Mathematicians, vol. II (2002, Beijing), Higher Education Press, Beijing, China, 2002, pp. 119–128.
- [CPS4] J.W. Cogdell and I.I. Piatetski–Shapiro, *Stability of Gamma Factors for $SO(2n + 1)$* , Manuscripta Math. **95** (1998), 437–461.
- [CKPSS1] J.W. Cogdell, H. Kim, I.I. Piatetski–Shapiro and F. Shahidi, *On lifting from classical groups to GL_N* , Publ. Math. IHES **93** (2001), 5–30.
- [CKPSS2] J.W. Cogdell, H.H. Kim, I.I. Piatetski–Shapiro and F. Shahidi, *Functoriality for the classical groups*, Publ. Math. IHES **99** (2004), 163–233.
- [CPSS1] J.W. Cogdell, I.I. Piatetski–Shapiro, and F. Shahidi, *Partial Bessel functions for quasi-split groups*, Automorphic Representations, L -functions and Applications: Progress and Prospects, Ohio State Univ. Math. Res. Inst. Publ. **11**, de Gruyter, Berlin, 2005, pp. 95–128.
- [CPSS2] J.W. Cogdell, I.I. Piatetski–Shapiro, and F. Shahidi, *Stability of γ -factors for quasi-split groups*, J. Inst. Math. Jussieu **7** (2008), 27–66.

- [CPSS3] J.W. Cogdell, I.I. Piatetski–Shapiro, and F. Shahidi, *Functoriality for quasisplit classical groups*, On Certain L -functions: Corresponding Proceedings on the Occasion of Freydoon Shahidi's 60th birthday, AMS/CMI, to appear.
- [CoSh] J. Cogdell and F. Shahidi, *Some generalized functionals and their Bessel functions*, in the volume in honor of S. Kudla, J. Cogdell, J. Fune, M. Rapoport and T. Yang, edit., Lectures in Mathematics and Higher Education, International Press, Beijing–Boston; in press.
- [De1] P. Deligne, *La conjecture de Weil. I*, Publ. Math. Inst. Hautes Études Sci. **43** (1974), 273–307.
- [De2] P. Deligne, *Les constantes des équation fonctionnelles des fonction L* , in Modular Functions of One Variable II, Proceedings of International Summer School, Antwerp, SLN 349, Springer–Verlag, Berlin–Heidelberg–New York, 1973.
- [DiMa] J. Dixmier and P. Malliavin, *Factorisations de fonctions et de vecteurs indéfiniment différentiables*, Bull. Sci. Math. **102** (1978), 305–330.
- [Fl] D. Flath, *Decomposition of representations into tensor products*, in Automorphic Forms and Automorphic Representations, Proc. Sympos. Pure Math. 33, I, AMS, Providence, RI, 1979, pp. 179–183.
- [FG] S. Friedberg and D. Goldberg, *On local coefficients for non-generic representations of some classical groups*, Compositio Math. **116** (1999), no. 2, 133–166.
- [FuHar] W. Fulton and J. Harris, *Representation Theory, A First Course*, GTM–RIM 129, Springer, 1991.
- [GGP] W.T. Gan, B.H. Gross and D. Prasad, *Restriction Problems for Classical Groups*, arxiv 0909.2999V1.
- [GJ] S. Gelbart and H. Jacquet, *A relation between automorphic representations of $GL(2)$ and $GL(3)$* , Ann. Scient. Éc. Norm. Sup. **11** (1978), 471–552.
- [GLa] S. Gelbart and E. Lapid, *Lower bounds for L -functions at the edge of the critical strip*, Amer. J. Math. **128** (2006), 619–638.
- [GLS] S. Gelbart, E. Lapid and P. Sarnak, *A new method for lower bounds of L -functions*, C.R. Math. Acad. Sci. Paris **339** (2004), 91–94.
- [GMi] S. Gelbart and S. Miller, *Riemann's zeta function and beyond*, Bulletin of AMS **41:1** (2003), 59–112.
- [GRoS] S. Gelbart, J. Rogawski and D. Soudry, *Endoscopy, theta liftings, and period integrals for unitary group in three variables*, Ann. of Math. **145(2)** (1997), no. 3, 419–476.
- [GSh1] S. Gelbart and F. Shahidi, *Analytic Properties of Automorphic L -functions*, Perspectives in Mathematics (J. Coates and S. Helgason, ed.), vol. 6, Academic Press, Inc., Harcourt Brace Jovanovich, Boston–San Diego–New York, 1988.
- [GSh2] S. Gelbart and F. Shahidi, *Boundedness of automorphic L -functions in vertical strips*, Journal of AMS **14** (2001), 79–107.
- [GGPS] I.M. Gelfand, M.I. Graev and I.I. Pyatetskii–Shapiro, *Representation Theory and Automorphic Functions*, Academic Press, Harcourt Brace Jovanovich, Publishers, 1990.
- [GRS] D. Ginzburg, S. Rallis, and D. Soudry, *Generic automorphic forms on $SO(2n + 1)$: functorial lift to $GL(2n)$, endoscopy, and base change*, IMRN, 2001:14, 2001, pp. 729–764.
- [GoSh1] D. Goldberg and F. Shahidi, *On the tempered spectrum of quasi-split classical groups*, Duke Math. J. **92** (1998), 255–294.
- [GoSh2] D. Goldberg and F. Shahidi, *On the tempered spectrum of quasi-split classical groups II*, Canad. J. Math. **53** (2001), 244–277.
- [GoSh3] D. Goldberg and F. Shahidi, *On the tempered spectrum of quasi-split classical groups III*, Forum Math.; in press.
- [GWa] R. Goodman and N.R. Wallach, *Whittaker vectors and conical vectors*, J. Functional Anal. **39** (1980), 199–279.
- [HC1] Harish–Chandra, *Automorphic forms on semisimple Lie groups*, SLN **62**, Berlin–Heidelberg–New York, 1968.
- [HC2] Harish–Chandra, *Collected papers, vol. IV*, Springer, Berlin, 1984.
- [HT] M. Harris and R. Taylor, *On the geometry and cohomology of some simple Shimura varieties*, Annals of Math. Studies, vol. 151, Princeton University Press, 2001.
- [Hash] M. Hashizume, *Whittaker models for real reductive groups*, Japan J. Math. **5** (1979), 349–401.

- [HMu] V. Heiermann and G. Muic, *The standard modules conjecture*, Math. Zeitschr. **255** (2007), 19–37.
- [HO] V. Heiermann and E. Opdam, *On the tempered L -function conjecture*, <http://arxiv.org/PS-cache/arxiv/pdf/0908/0908.0699V4.pdf>.
- [He1] G. Henniart, *La conjecture de Langlands locale pour $GL(3)$* , Mémoires de la Soc. Math. de France **11/12** (1984), 1–186.
- [He2] G. Henniart, *Une preuve simple des conjectures de Langlands pour $GL(n)$ sur un corps p -adique*, Invent. Math. **139** (2000), 439–455.
- [He3] G. Henniart, *Progrès récents en fonctorialité de Langlands*, Seminaire Bourbaki (Juin 2001); Exposé 890, 890–1 to 890–21.
- [HeLo] G. Henniart and L. Lomeli, *Local-to-global extensions for GL_n in non-zero characteristic: a characterization of $\gamma_F(s, \pi, Sym^2, \psi)$ and $\gamma_F(s, \pi, \Lambda^2, \psi)$* , Amer. J. Math.; in press.
- [Hu] J.E. Humphreys, *Introduction to Lie Algebras and Representations Theory*, Springer-Verlag, GTM9, 1980.
- [Iw] H. Iwaniec, *Spectral methods of automorphic forms*, 2nd ed., Graduate Studies in Mathematics, vol. 53, American Mathematical Society, Providence, RI, 2002.
- [IwSa] H. Iwaniec and P. Sarnak, *Perspectives on the analytic theory of L -functions*, Geom. Funct. Anal. **2000 (Special Issue)**, 705–741.
- [J] H. Jacquet, *Fonctions de Whittaker associées aux groupes de Chevalley*, Bull. Soc. Math. France **95** (1967), 243–309.
- [JLa] H. Jacquet and R.P. Langlands, *Automorphic Forms on $GL(2)$* , SLN, Vol. 114, Springer-Verlag, Berlin-Heidelberg-New York, 1970.
- [JPSS] H. Jacquet, I.I. Piatetski-Shapiro, and J.A. Shalika, *Rankin-Selberg convolutions*, Amer. J. Math. **105:2** (1983), 367–464.
- [JPSS2] H. Jacquet, I.I. Piatetski-Shapiro, and J.A. Shalika, *Relèvement cubique non normal*, C.R. Acad. Sci. Paris Sér. I Math. **292** (1981), 567–571.
- [JS1] H. Jacquet and J.A. Shalika, *On Euler products and the classification of automorphic representations, I*, Amer. J. Math. **103:3** (1981), 499–558.
- [JS2] H. Jacquet and J.A. Shalika, *On Euler products and the classification of automorphic representations, II*, Amer. J. Math. **103** (1981), 777–815.
- [JiSo] D. Jiang and D. Soudry, *Generic representations and local Langlands reciprocity law for p -adic SO_{2n+1}* , Contributions to Automorphic Forms, Geometry and Number Theory (Shalikafest 2002) (H. Hida, D. Ramakrishnan, and F. Shahidi, ed.), Johns Hopkins University Press, Baltimore, 2004, pp. 457–520.
- [KeSh] D. Keys and F. Shahidi, *Artin L -functions and normalization of intertwining operators*, Ann. Scient. Éc. Norm. Sup. **21** (1988), 67–89.
- [KLS] C. Khare, M. Larsen and G. Savin, *Functoriality and the inverse Galois problem*, Compos. Math. **144** (2008), 541–564.
- [K1] H.H. Kim, *Langlands-Shahidi method and poles of automorphic L -functions: Application to exterior square L -functions*, Can. J. Math. **51** (1999), 835–849.
- [K2] H.H. Kim, *Langlands-Shahidi method and poles of automorphic L -functions II*, Israel J. Math. **117** (2000), 261–284.
- [K3] H.H. Kim, *On local L -functions and normalized intertwining operators*, Can. J. Math. **57** (2005), no. 3, 535–597.
- [K4] H.H. Kim, *Functoriality for the exterior square of GL_4 and symmetric fourth of GL_2* , Journal of AMS **16** (2002), 139–183.
- [K5] H.H. Kim, *Automorphic L -functions*, in Lectures on Automorphic L -functions, Fields Institute Monographs, AMS, 2004, pp. 99–201.
- [K6] H.H. Kim, *Langlands-Shahidi method and poles of automorphic L -functions, III; Exceptional groups*, J. Number Theory **128** (2008), 354–376.
- [KKi] H.H. Kim and W. Kim, *On local L -functions and normalized intertwining operators II; quasi-split groups*, to appear in Shahidi’s birthday conference volume.
- [KK] H.H. Kim and M. Krishnamurthy, *Stable base change lift from unitary groups to GL_n* , Int. Math. Res. Pap. **IMRP** (2005), no. 1, 1–52.
- [KSa] H.H. Kim and P. Sarnak, *Refined estimates towards the Ramanujan and Selberg conjectures*, Appendix 2 to [K4].

- [KSh1] H.H. Kim and F. Shahidi, *Symmetric cube L -functions for GL_2 are entire*, Ann. of Math. **150** (1999), 645–662.
- [KSh2] H.H. Kim and F. Shahidi, *Factorial products for $GL_2 \times GL_3$ and the symmetric cube for GL_2* , Annals of Math. **155** (2002), 837–893.
- [KSh3] H.H. Kim and F. Shahidi, *Cuspidality of symmetric powers with applications*, Duke Math. J. **112** (2002), 177–197.
- [KSh4] H.H. Kim and F. Shahidi, *On the holomorphy of certain L -functions*, in “Contributions to Automorphic Forms, Geometry and Arithmetic,” in honor of Joseph Shalika’s 60th Birthday (H. Hida, D. Ramakrishnan and F. Shahidi, ed.), The Johns Hopkins University Press, Baltimore–London, 2004, pp. 561–572.
- [KSh5] H.H. Kim and F. Shahidi, *On simplicity of poles of automorphic L -functions*, Journal of Ramanujan Mathematical Society **19** (2004), 177–197.
- [KSh6] H.H. Kim and F. Shahidi, *Holomorphy of the 9th symmetric power L -functions for $Re(s) > 1$* , IMRN (2006); Art. ID59326, 7 pp.
- [Ki] W. Kim, *Square Integrable Representations and the Standard Module Conjecture for General Spin Groups*, Can. J. Math. **61** (2009), no. 3, 617–640.
- [Kn1] A.W. Knap, *Representation Theory of Semisimple Groups*, Princeton University Press, Princeton, 1986.
- [Kn2] A.W. Knap, *Local Langlands Correspondence: The Archimedean Case*, Motives (U. Jannsen, S. Kleiman and J-P. Serre, ed.), Proc. Sympos. Pure Math., AMS, 55, Part 2, 1994, pp. 393–410.
- [KnSt1] A.W. Knap and E.M. Stein, *Intertwining operators for semisimple groups*, Ann. of Math. **93** (1971), 489–578.
- [KnSt2] A.W. Knap and E.M. Stein, *Intertwining operators for semisimple groups II*, Invent. Math. **60** (1980), 9–84.
- [KnWa] A.W. Knap and N.R. Wallach, *Szegő kernels associated with discrete series*, Invent. Math. **34** (1976), 163–200.
- [KnZ] A.W. Knap and G.J. Zuckerman, *Classification of irreducible tempered representations of semisimple groups*, Ann. of Math. **116** (1982), 389–455.
- [Kon] T. Konno, *Twisted endoscopy and the generic Packet conjecture*, Israel J. Math. **129** (2002), 253–289.
- [Kos] B. Kostant, *On Whittaker vectors and representation theory*, Invent. Math. **48** (1978), 101–184.
- [KS] R. Kottwitz and D. Shelstad, *Foundations of Twisted Endoscopy*, Astérisque **255** (1999), 1–190.
- [Kud] S.S. Kudla, *The local Langlands correspondence: The non-archimedean case*, Motives (U. Jannsen, S. Kleiman and J-P. Serre, ed.), Proc. Sympos. Pure Math., AMS, 55, Part 2, 1994, pp. 365–391.
- [Ku] P. Kutzko, *The Langlands conjecture for GL_2 of a local field*, Ann. of Math. **112** (1980), 381–412.
- [L] J-P. Labesse, *Cohomologie, L -groupes et fonctorialité*, Comp. Math. **55** (1984), 163–184.
- [LLa] J-P. Labesse and R.P. Langlands, *L -indistinguishability for $SL(2)$* , Can. J. Math. **31** (1979), 726–785.
- [Laf] L. Lafforgue, *Chtoucas de Drinfeld et correspondance de Langlands*, Invent. Math. **147:1** (2002), 1–241.
- [Lai] K.F. Lai, *On the Tamagawa number of quasisplit groups*, thesis, Yale University (1974).
- [La1] R.P. Langlands, *Euler Products*, Yale University Press, 1971.
- [La2] R.P. Langlands, *On the Functional Equations Satisfied by Eisenstein Series*, Lecture Notes in Math., vol. 544, Springer–Verlag, 1976.
- [La3] R.P. Langlands, *Problems in the theory of automorphic forms*, in Lecture Notes in Math. 170, Springer–Verlag, Berlin–Heidelberg–New York, 1970, pp. 18–86.
- [La4] R.P. Langlands, *On the functional equation of the Artin L -functions*, Yale University, Mimeographed Notes.
- [La5] R.P. Langlands, *On Artin’s L -functions*, Rice Univ. Studies 56, Houston, TX, 1970.
- [La6] R.P. Langlands, *On the classification of irreducible representations of real algebraic groups*, Representation Theory and Harmonic Analysis on Semisimple Lie Groups

- (P.J. Sally, Jr. and D.A. Vogan, ed.), vol. 31, Math. Surveys and Monographs, A.M.S., 1989, pp. 101–170.
- [La7] R.P. Langlands, *Automorphic representations, Shimura varieties, and motives. Ein Märchen*, Proc. Sympos. Pure Math., vol. 33, Part 2, 1979, pp. 205–246.
- [La8] R.P. Langlands, *Base change for $GL(2)$* , Ann. of Math. Studies **96**, Princeton Univ. Press, Princeton, NJ, 1980.
- [La9] R.P. Langlands, *Representations of Abelian Algebraic Groups*, Olga Taussky–Todd Memorial Issue, Pacific J. Math., 1997, pp. 231–250.
- [Le] B. Ya. Levin, *Lectures on Entire Functions*; In Collaboration with Yu. Lyubarskii, M. Sodin, and V. Tkachenko, Trans. Math. Mono. **150**; AMS (1996).
- [Lo] L. Lomeli, *Functoriality for the classical groups over function fields*, International Mathematics Research Notices (2009); doi: 10.1093/imrn/rnp089, 65 pages.
- [LRSa] W. Luo, Z. Rudnick and P. Sarnak, *On the generalized Ramanujan Conjecture for $GL(n)$* , in Proc. Sympos. Pure Math., vol. 66, 1999, pp. 301–310.
- [Mac] I.G. Macdonald, *Spherical functions on a p -adic Chevalley group*, Bull. Amer. Math. Soc. **74** (1968), 520–525.
- [MW1] C. Moeglin and J.-L. Waldspurger, *Le spectre résiduel de $GL(n)$* , Ann. Sci. École Norm. Sup. **22** (1989), 605–674.
- [MW2] C. Moeglin and J.-L. Waldspurger, *Spectral decomposition and Eisenstein series*, Cambridge Tracts in Math. **113**, Cambridge University Press, 1995.
- [MuSh] G. Muić and F. Shahidi, *Irreducibility of standard representations for Iwahori–Spherical Representations*, Math. Ann. **312** (1998), 151–165.
- [Mü] W. Müller, *The trace class conjecture in the theory of automorphic forms*, Ann. of Math. **130** (1989), 473–529.
- [Mur] M. Ram Murty, *Applications of symmetric power L -functions*, in Lectures on Automorphic L -functions, Fields Institute Monographs, AMS, 2004, pp. 205–283.
- [Ra] D. Ramakrishnan, *Modularity of the Rankin–Selberg L -series, and multiplicity one for $SL(2)$* , Ann. of Math. **152** (2000), 45–111.
- [RaSh] D. Ramakrishnan and F. Shahidi, *Siegel modular forms of genus 2 attached to elliptic curves*, Math. Res. Lett. **14** (2007), 315–332.
- [RaWa] D. Ramakrishnan and S. Wang, *A cuspidality criterion for the functorial product on $GL(2) \times GL(3)$ with a cohomological application*, IMRN **27** (2004), 1355–1394.
- [Ro] F. Rodier, *Whittaker models for admissible representations of reductive p -adic split groups*, in Harmonic Analysis on Homogeneous Spaces, Proc. Sympos. Pure Math., 24, AMS, Providence, RI, 1973, pp. 425–430.
- [Sat1] I. Satake, *Theory of spherical functions on reductive algebraic groups over p -adic fields*, IHES Publ. Math. **18** (1963), 1–69.
- [Sat2] I. Satake, *Classification Theory of Semi-Simple Algebraic Groups*, Marcel Dekker, New York, 1971.
- [Sch] G. Schiffmann, *Intégrales d’entrelacement et fonctions de Whittaker*, Bull. Soc. Math. France **99** (1971), 3–72.
- [Sel] A. Selberg, *On the estimation of Fourier coefficients of modular forms*, Proc. Sympos. Pure Math. **8** (1965), 1–15.
- [Se] J-P. Serre, *Algèbres de Lie Semi-Simple Complexes*, W.A. Benjamin, 1966.
- [Se2] J-P. Serre, *Abelian l -adic Representations and Elliptic Curves*, W.A. Benjamin, New York, 1968.
- [Sh1] F. Shahidi, *Functional equation satisfied by certain L -functions*, Comp. Math. **37** (1978), 171–208.
- [Sh2] F. Shahidi, *Whittaker models for real groups*, Duke Math. **47** (1980), 99–125.
- [Sh3] F. Shahidi, *On certain L -functions*, Amer. J. Math. **103(2)** (1981), 297–355.
- [Sh4] F. Shahidi, *Some results on L -indistinguishability for $SL(r)$* , Canad. J. Math. **35** (1983), 1075–1109.
- [Sh5] F. Shahidi, *Fourier transforms of intertwining operators and Plancherel measures for $GL(n)$* , Amer. J. Math. **106** (1984), 67–111.
- [Sh6] F. Shahidi, *Local coefficients as Artin factors for real groups*, Duke Math. J. **52** (1985), 973–1007.
- [Sh7] F. Shahidi, *On the Ramanujan conjecture and finiteness of poles for certain L -functions*, Annals of Math. **127** (1988), 547–584.

- [Sh8] F. Shahidi, *A proof of Langlands Conjecture on Plancherel measures; Complementary series for p -adic groups*, *Annals of Math.* **132** (1990), 273–330.
- [Sh9] F. Shahidi, *On multiplicativity of local factors*, in *Festschrift in honor of I.I. Piatetski-Shapiro, Part II*, *Israel Mathematical Conference Proceedings*, vol. 3, 1990, pp. 279–289.
- [Sh10] F. Shahidi, *Langlands’ Conjecture on Plancherel measures for p -adic groups*, in *Harmonic Analysis on Reductive Groups* (W. Barker and P. Sally, ed.), *Progress in Mathematics*, Birkhäuser, 1991, pp. 277–295.
- [Sh11] F. Shahidi, *Twisted endoscopy and reducibility of induced representations for p -adic groups*, *Duke Math.* **66** (1992), 1–41.
- [Sh12] F. Shahidi, *The notion of norm and the representation theory of orthogonal groups*, *Invent. Math.* **119** (1995), 1–36.
- [Sh13] F. Shahidi, *Intertwining operators, L -functions, and representation theory*, *Lecture notes of the Eleventh KAIST Mathematics Workshop 1996* (J.K. Koo, ed.), KAIST, Taejon, Korea, 1998, pp. 1–63.
- [Sh14] F. Shahidi, *On non-vanishing of twisted symmetric and exterior square L -functions for $GL(n)$* , in volume dedicated to Olga Taussky-Todd (M. Aschbacher, D. Blasius, and D. Ramakrishnan, ed.), *Pacific J. Math.*, International Press, 1998, pp. 311–322.
- [Sh15] F. Shahidi, *Poles of intertwining operators via endoscopy; the connection with prehomogeneous vector spaces*, with an appendix “Basic endoscopic data” by D. Shelstad, dedicated to the memory of Magdy Asem, *Comp. Math.* **120** (2000), 291–325.
- [Sh16] F. Shahidi, *Twists of a general class of L -functions by highly ramified characters*, *Canad. Math. Bull.* **43** (2000), 380–384.
- [Sh17] F. Shahidi, *Automorphic L -functions and Functoriality*, *Proceedings of the International Congress of Mathematicians*, vol. II (2002, Beijing), Higher Education Press, Beijing, China, pp. 655–666.
- [Sh18] F. Shahidi, *Local coefficients as Mellin transforms of Bessel functions; Towards a general stability*, *International Mathematics Research Notices (IMRN)* **2002** (2002), no. 39, 2075–2119.
- [Sh19] F. Shahidi, *On the Ramanujan conjecture for quasisplit groups*, *Asian Jour. of Math.*; (Volume in the Memory of Armand Borel) **8** (2004), 813–836.
- [Sh20] F. Shahidi, *Langlands–Shahidi Method*, in “Automorphic Forms and Applications” (P. Sarnak and F. Shahidi, ed.), *IAS/Park City Mathematics Series*, vol. 12, AMS/IAS, 2007, pp. 299–330.
- [Sh21] F. Shahidi, *Complexity of group actions and stability of root numbers*, with an appendix by W. Kuo; in *Automorphic Forms and L -functions II, Local Aspects*, the volume in honor of S. Gelbart’s 60th Birthday (D. Ginzburg, E. Lapid and D. Soudry, ed.); *Contemporary Mathematics* 489, *Israel Mathematical Conference Proceedings*, 2009, pp. 259–276; (Appendix: 277–283).
- [Sh22] F. Shahidi, *L -functions and poles of intertwining operators*; Appendix to: “Residues of intertwining operators for classical groups” by S. Spallone, *International Mathematics Research Notices (IMRN)* 2008 (2008), article ID rnn 095, 13 pages.
- [Sh23] F. Shahidi, *Arthur Packets and the Ramanujan Conjecture*, preprint.
- [ShSp] F. Shahidi and S. Spallone, *Residues of intertwining operators for SL_6^* as character identities*, *Compositio Math.*, to appear.
- [S] J.A. Shalika, *The multiplicity one for GL_n* , *Ann. of Math.* **100** (1974), 171–193.
- [S2] J.A. Shalika, *A theorem on semisimple p -adic groups*, *Annals of Math.* **95** (1972), 226–242.
- [Shi] G. Shimura, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton University Press, 1994.
- [Sil1] A. Silberger, *Introduction to harmonic analysis on reductive p -adic groups*, *Mathematical Notes*, vol. 23, Princeton University Press, Princeton, NJ, 1979.
- [Sil2] A. Silberger, *The Langlands quotient theorem for p -adic groups*, *Math. Ann.* **236** (1978), 95–104.
- [Sil3] A. Silberger, *Special representations of reductive p -adic groups are not integrable*, *Ann. of Math.* **111** (1980), 571–587.
- [So1] D. Soudry, *Rankin–Selberg convolutions for $SO_{2l+1} \times GL_n$: local theory*, *Mem. Amer. Math. Soc.* **105**, no. **500** (1993), vi+100.

- [So2] D. Soudry, *On the Archimedean theory of Rankin–Selberg convolutions for $SO_{2l+1} \times GL_n$* , Ann. Sci. Ecole Norm. Sup. (4) **28**, no. 2 (1995), 161–224.
- [So3] D. Soudry, *On Langlands functoriality from classical groups to GL_n , Formes Automorphes (I)*, Asterisque **298** (2005), 335–390.
- [Sp1] S. Spallone, *Residues of intertwining operators for classical groups*, Int. Math. Res. Not. (IMRN) (2008); Art. ID rnm 056, 37 pages.
- [Spr1] T.A. Springer, *Reductive groups*, in Automorphic Forms and Automorphic Representations, Proc. Sympos. Pure Math. 33, I, AMS, Providence, RI, 1979, pp. 3–27.
- [Spr2] T.A. Springer, *Linear Algebraic Groups*, PM9, Birkhäuser, Boston, 1981.
- [Sun] R. Sundaravaradhan, *Some structural results for the stability of root numbers*, Int. Math. Res. Not. IMRN **2008**, no. 2; Art. ID rnm141, 22 pp.
- [Sz] D. Szpruch, *On some applications of the Langlands–Shahidi method to the metaplectic group*, thesis, Univ. of Tel Aviv (2009).
- [T] J. Tate, *Number theoretic background*, Proc. Sympos. Pure Math., AMS **33**, II, 1979, pp. 3–26.
- [Tay] R. Taylor, *Automorphy for some l -adic lifts of automorphic mod l Galois representations II*, Publ. Math. IHES **108** (2008), 183–239.
- [Ti] J. Tits, *Reductive groups over local fields*, in Automorphic Forms and Automorphic Representations, Proc. Sympos. Pure Math. 33; I, AMS, Providence, RI, 1979, pp. 29–69.
- [Tit] E.C. Titchmarsh, *The Theory of Functions*, Oxford Science Publications, Second Edition, Oxford University Press, Oxford, New York–Toronto, 1991.
- [Va] V.S. Varma, *Descent and the generic packet conjecture*, thesis, Purdue University (2009).
- [Vi] M.-F. Vignéras, *Correspondances entre représentations automorphes de $GL(2)$ sur une extension quadratique de $GSp(4)$ sur \mathbf{Q} , conjecture locale de Langlands pour $GSp(4)$* , Contemp. Math. **53** (1986), 463–527.
- [V] D. Vogan, *Gelfand–Kirillov dimension for Harish–Chandra modules*, Invent. Math. **48** (1978), 75–98.
- [Wa1] N.R. Wallach, *Real reductive groups, I & II*, Academic Press, Pure and Applied Mathematics, vol. 132 & 132 II, 1988 & 1992.
- [Wa2] N.R. Wallach, *Asymptotic expansions of generalized matrix entries of representations of real reductive groups*, in Lie Group Representations I, SLN **1024**, Springer–Verlag, 1983, pp. 287–369.
- [W] G. Warner, *Harmonic Analysis on Semi–Simple Lie Groups I*, Grundlehren **188**, Springer Verlag, New York, 1972.
- [We] A. Weil, *Adeles and Algebraic Groups*, Progress in Math., Vol. 23, Birkhäuser, Boston–Basel–Stuttgart, 1982.
- [Yo] K. Yoshida, *Functional Analysis*, Springer–Verlag, Berlin–Heidelberg–New York, 1971.
- [Yu] X. Yu, *Centralizers and twisted centralizers: application to intertwining operators*, Canad. J. Math. **58** (2006), 643–672.
- [Z] Y. Zhang, *The holomorphy and nonvanishing of normalized local intertwining operators*, Pacific J. Math. **180** (1997), 385–398.

Subject Index

- (local) γ -factors, 141
- χ -Fourier coefficient, 130
- χ -generic representation, 136
- χ -generic Whittaker functional, 46
- χ -Whittaker functional, 76
- χ_M -generic subquotients, 76
- Γ -diagram, 182
- Γ -functions, 158
- γ -functions, 133, 140
- Γ -orbits, 8

- absolutely simple, 182
- absorbing set, 79
- adelization, 11
- adjoint, 13
- adjoint action, 118
- adjoint action of ${}^L M$ on ${}^L \mathfrak{n}$, 142
- adjoint action of \hat{T} , 85
- adjoint of intertwining operator, 104
- admissible homomorphism, 137
- admissible representation, 27
- algebra of bi-invariant differential operators, 110
- analytic functions of finite order, 162
- archimedean Hecke eigenvalues, 178
- Arthur's A -packet conjecture, 180
- Artin L -function and root number, 167
- Artin L -functions, 138
- Artin root numbers, 135, 141
- associate, 81
- asymptotic expansions, 177

- backward lifting, 180
- balanced set, 79
- Banach representation, 82
- based root datum, 12
- Bessel functions, 177
- Bessel models, 180
- Borel subgroup, 7, 143
- boundedness in vertical strips, 156
- Bourbaki labeling, 181

- canonical Whittaker functional, 99
- Cartan subgroup, 179

- Casselman's subrepresentation theorem, 74
- central character, 169
- character identities of Labesse-Langlands, 179
- character modules, 23
- classical groups, 175
- closed half plane, 157
- commutator subgroup of W_k , 137
- compatible, 51, 99
- complementary series, 149, 151
- complete, 79
- computation of the constant term, 111
- conductor, 163
- connected reductive group, 7
- connection with L -functions, 115
- constant term, 111, 113
- converse theorems, 167, 169, 170
- crude functional equations, 129
- cuspidal representations, 115, 118, 129
- cyclic base change, 170

- derived group, 181
- discrete series, 146, 149
- discriminant function, 179
- double coset, 112
- Dynkin diagrams, 181

- Eisenstein series, 107, 161
- endoscopy, 179
- equality up to a monomial, 148
- examples of multiplicativity, 142
- explicit formulas for local coefficients, 131
- exterior square representation, 143, 177

- factorization of intertwining operators, 84
- finite order, 156
- finiteness of poles, 153, 154
- Fourier coefficient, 161
- Fourier coefficients of Eisenstein series, 121
- Fourier transforms, 133
- Fréchet representation, 82
- Fréchet space, 49
- function fields, 180

- functional equations, 121, 129, 142, 147, 148
- functorial transfer, 167, 174
- Functoriality, 167
- fundamental weights, 181
- \hat{G} -semisimple-conjugacy class, 175
- Gelbart-Jacquet lift, 174
- general spin groups, 176
- generic characters, 45, 122, 143
- generic representations, 45
- generic spectrum, 180
- Gindikin-Karpelevich, 84
- global Hecke algebra, 33
- global intertwining operators, 107, 109
- global partial L -functions, 33
- globally χ -generic, 47
- globally generic representations, 46, 143, 147
- globally induced representation, 116
- größencharacter, 169
- GSpin groups, 167
- H^1 , 12, 95, 97
- \mathcal{H}_S^0 , 169
- $\mathcal{H}_S^m(\chi)$, 169, 171
- half strips of finite width, 156
- Harish-Chandra's γ -constant, 105
- Hecke algebra $\mathcal{H}(G(k), K)$, 27
- Hecke eigenvalues, 167, 177
- Hecke L -function, 136
- Hecke-Tate γ -function, 133
- highest weight, 37
- highest weights, 181
- highly ramified character, 166, 171
- highly ramified twists, 163
- Hilbert modular form, 177
- holomorphy, 154
- hyperbolic Laplacian, 177
- hyperbolic lattice point problem, 178
- hyperbolic Riemann surface, 177
- hyperspecial, 140
- indecomposable, 137
- indecomposable constituent, 139
- index, 141
- induced representations, 49, 108, 122
- induced Whittaker functional, 50
- induction step, 118
- intertwining operators, 63, 66, 81, 83, 90, 99, 117, 149
- inverse Frobenius, 138
- irreducible admissible Banach representation, 49
- isobaric automorphic representation, 175
- isobaric sum, 170, 177
- Iwahori fixed vector, 137, 138
- Iwahori subgroup, 60
- Iwasawa decomposition, 32
- Jacquet integral, 125, 126, 158
- Jacquet's functional equation, 68
- k -anisotropic, 16
- K -finite function, 128
- k -rational characters, 23
- k -splitting, 129, 153
- K_∞ -finite, 110
- K -types, 151
- L -functions, 95, 121, 129, 141, 144, 168
- L -groups, 12, 117
- Langlands classification, 87, 144, 145
- Langlands decomposition, 73, 74
- Langlands functoriality conjecture, 153, 175
- Langlands functoriality principle, 169
- Langlands λ -function, 134
- Langlands lemma, 84
- Langlands quotient, 149, 151
- Laplacian eigenvalues of Maass forms, 167, 177
- Levi subgroups, 8, 14
- local coefficients, 99, 129, 133, 136, 148, 159
- local coefficients for real groups, 134
- local functorial transfer, 179
- local Langlands correspondence, 179
- local Langlands L -function, 31
- local root numbers, 129
- locally χ -generic, 47
- locally convex space, 79
- locally generic cuspidal representation, 180
- long element of $W(A_0, M)$, 125
- Maass forms, 177
- Maass-Selberg relation, 157
- Macdonald's formulas, 34
- Matsaev, 161
- maximal anisotropic subtorus, 7
- maximal compact subgroups, 115
- maximal parabolic, 9
- maximal split subtorus, 7, 16
- meromorphic continuation, 130
- meromorphy of individual $L^S(s, \pi, r_i)$, 118
- metaplectic covering groups, 180
- metaplectic Eisenstein series, 180
- minimal parabolic subgroup, 8
- monomial, 174
- multiple Dirichlet series, 180
- multiplicativity, 102, 129, 140, 145, 146
- multiplicativity of local coefficients, 101
- nilpotent endomorphism, 137
- non-constant term, 122
- non-cyclic base change, 170
- non-monomial cuspidal representation, 174
- non-restricted simple roots, 132
- non-trivial additive character, 129
- non-trivial character, 121
- non-vanishing, 154

- non-vanishing of L -functions, 127
- normalization of intertwining operators, 150
- normalized intertwining operators, 179
- normalized operators, 150, 153
- normalizing factor, 150, 153
- orbital integrals, 177
- ordered basis, 134
- p -adic integration and measures, 180
- parabolic subgroup of ${}^L G$, 14
- parabolic subgroups, 8, 14
- parabolic subgroups of L -groups, 14
- partial L -function, 43
- Phragmén-Lindelöf's theorem, 156
- Plancherel measures, 105, 151
- Poincaré series, 143
- poles of intertwining operators, 149, 178
- positive definite, 151
- positive or negative semi-definite, 151
- positive roots, 8
- positive Weyl chamber, 158, 159
- principal series, 85
- quasi-tempered representation, 145
- quasisplit connected reductive group, 118
- quasisplit group, 7
- quasisplit special orthogonal groups, 18
- quasisplit unitary group in 3-variables, 17
- radical, 7
- Ramanujan conjecture, 177, 180
- Ramanujan-Selberg estimates, 177
- ramified, 149
- rank one group, 132
- Rankin product L -functions, 144
- Rankin-Selberg L -functions, 167
- Rankin-Selberg method, 129, 143
- rational character, 163
- real poles, 155
- reduced roots, 88
- reducibility, 149, 178
- reducibility points, 152
- reductive groups, 7
- reflection representatives, 132
- relevant, 14
- restriction of scalars, 95, 132
- root numbers, 138, 144, 145
- root system, 8
- Satake isomorphisms, 23, 27
- Satake transformation, 28
- Sato-Tate conjecture, 173
- Schur's lemma, 105
- self dual measure, 133, 136
- self-associate, 11, 148, 164
- seminorms, 79, 159
- semisimple conjugacy class, 177
- Shalika germs, 177
- simply connected, 13, 181
- smooth vectors, 49
- special maximal compact subgroup, 143
- special orthogonal or symplectic group, 176
- spherical function, 34, 125, 158
- spherical vector, 116
- split component, 108
- split component of a reductive group, 9
- splitting, 12, 131
- splitting defined over k , 121
- splitting field, 131
- stability of local L -functions, 176
- stability of root numbers, 176
- standard intertwining operator, 150
- standard maximal parabolic subgroup, 111
- standard module, 74
- standard parabolic subgroups, 81
- supercuspidal, 144
- supercuspidal χ -generic representation, 148
- supercuspidal inducing data, 149
- $\mathrm{Sym}^m \mathbb{C}^2$, 172
- symmetric power representations, 172
- symmetric tensors, 172
- Tamagawa number, 133
- tempered, 146
- tempered L -packets conjecture, 180
- tempered representation, 144
- third symmetric powers for $GL(2)$, 172
- tori, 15
- truncation, 157
- twisted Jacquet module, 52
- unipotent radical, 7, 81
- unitary, 150
- unitary representation, 49, 82
- universal enveloping algebra, 46, 110
- universal projection, 85
- unramified, 7
- unramified character, 26
- unramified characters and representations, 23
- unramified group, 27
- unramified principal series, 85
- unramified simply connected groups of rank one, 60
- unramified Whittaker Functions, 60
- vertical strip of finite width, 168
- Weil group, 135
- Weil-Deligne group, 137, 167
- Weyl group, 8, 132
- Weyl set, 109
- Weyl-translate, 159
- Whittaker Fourier coefficients of Eisenstein series, 121
- Whittaker functional, 122, 127

Whittaker functions, 47

Whittaker functions for principal series, 74

Whittaker models, 46, 47, 124

\mathcal{Z}_∞ -finite, 110

$\mathcal{Z}_{M,\infty}$ -finite, 115

This book presents a treatment of the theory of L -functions developed by means of the theory of Eisenstein series and their Fourier coefficients, a theory which is usually referred to as the Langlands–Shahidi method. The information gathered from this method, when combined with the converse theorems of Cogdell and Piatetski-Shapiro, has been quite sufficient in establishing a number of new cases of Langlands functoriality conjecture; at present, some of these cases cannot be obtained by any other method. These results have led to far-reaching new estimates for Hecke eigenvalues of Maass forms, as well as definitive solutions to certain problems in analytic and algebraic number theory.

This book gives a detailed treatment of important parts of this theory, including a rather complete proof of Casselman–Shalika’s formula for unramified Whittaker functions as well as a general treatment of the theory of intertwining operators. It also covers in some detail the global aspects of the method as well as some of its applications to group representations and harmonic analysis.

This book is addressed to graduate students and researchers who are interested in the Langlands program in automorphic forms and its connections with number theory.

ISBN 978-0-8218-4989-7



9 780821 849897

COLL/58



For additional information
and updates on this book, visit

www.ams.org/bookpages/coll-58

AMS on the Web
www.ams.org