

# CONTEMPORARY MATHEMATICS

330

## Recent Advances in Scientific Computing and Partial Differential Equations

International Conference on the Occasion of  
Stanley Osher's 60th Birthday

December 12–15, 2002

Hong Kong Baptist University, Hong Kong

S. Y. Cheng

C.-W. Shu

T. Tang

Editors



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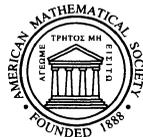
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## Preface

Over the past decade there have been very rapid developments of efficient algorithms for scientific computing and related investigations of mathematical issues for partial differential equations and image processing. As a result, many problems in diverse application fields such as fluid dynamics, image processing, computer vision, and computer graphics in the entertainment industry can now be routinely simulated to high resolution. Stanley Osher has been in the forefront of many of these developments, and has published many influential original works on high resolution shock capturing schemes, level set methods, applications to multi-phase flows, computer vision, TV based image restoration, just to name a few areas.

On the occasion of the 60th birthday of Stanley Osher, the International Conference on Scientific Computing and Partial Differential Equations was held in Lam Woo Conference Center, Hong Kong Baptist University, Hong Kong from December 12-15, 2002. The main goal of this conference is to review recent developments and to explore exciting new directions in scientific computing and partial differential equations for time dependent problems and its interaction with other fields such as image processing, computer vision and graphics. An emphasis of this conference is the strong interaction of significant mathematics with advanced algorithms applicable to real-world applications.

The conference attracted a number of leading scientists in computational PDEs including Yann Brenier (CNRS Nice & Paris 6, France), Tony Chan (UCLA, USA), Chia-Kun Chu (Columbia, USA), Bjorn Engquist (UCLA & Princeton, USA), Yoshikazu Giga (Hokkaido, Japan), Roland Glowinski (Houston, USA), David Gottlieb (Brown, USA), Daniel Joseph (Minnesota, USA), Fanghua Lin (Courant Institute, NYU, USA), Barry Merriman (UCLA, USA), Guillermo Sapiro (Minnesota, USA), Zhong-Ci Shi (Chinese Academy of Sciences, China), Endre Suli (Oxford, UK), Dimitri Vvedensky (Imperial College, London, UK), Zhouping Xin (Chinese University, Hong Kong) and Takashi Yabe (Tokyo Institute of Technology, Japan). It also drew an international audience of 100. The detailed information including invited speakers, organizing committee and conference programs can be found at <http://www.math.hkbu.edu.hk/SCPDE02>.

We are very grateful for the generous donations to support this conference given by The Croucher Foundation, The Epson Foundation, Hong Kong Pei Hua Education Foundation, K.C. Wong Education Foundation, The Hong Kong Mathematical Society, Hong Kong Baptist University, Peking University, and PKU-HKBU Joint Research Institute for Applied Mathematics ([www.math.hkbu.edu.hk/JRIAM](http://www.math.hkbu.edu.hk/JRIAM)).

A number of staff at Hong Kong Baptist University worked together to handle the detailed arrangements. We would especially like to thank Fred Hickernell,

Elsa Fong, Tammy Lam, C. W. Yeung, Suan Choi and Amy Lee. Their help has been indispensable. We appreciate their assistance in making the conference organization a success.

These conference proceedings were refereed. We would like to thank all referees for their support. We thank Tammy Lam for the considerable work she put into producing the final layout of the proceedings. We also thank Dr. Barry Merriman for writing a foreword for this special volume.

*S. Y. Cheng, Hong Kong Univ. of Science and Technology*

*C.-W. Shu, Brown University*

*T. Tang, Hong Kong Baptist University*

*Editors*

*March 2003*

## What's The Occasion? Stan Osher's 60th Birthday!

It is a pleasure to dedicate this special issue to Professor Stanley J. Osher, in honor of his contributions to mathematics, and on the occasion of his 60th Birthday. This volume represents the collective appreciation of Stan's colleagues, students, and the many scientists and engineers that have benefited from his work.

By simple measures, Stan has had an incredibly productive scientific career, authoring over 150 scientific papers, mentoring more than 30 PhD students, and co-founding three companies based on his research. This work is even more impressive for its content, which includes three major innovations in computational mathematics: the ENO methods for hyperbolic conservation laws, PDE-based image processing, and the Level Set Method for moving interface problems.

On this special occasion it seems fitting to look beyond these widely known accomplishments, to provide a "Brief History of Stan". Stanley Joel Osher was born to poor, second-generation Jewish immigrant parents—neither of which finished high school—in Brooklyn, New York, on April 24, 1942. Stan often remarks that he shares this birth date and place with singer Barbara Streisand. And, not to be outdone, 60 years later he lives in the same neighborhood of Los Angeles as Streisand, as well. Stan grew up in the stereotypical rough Brooklyn neighborhood—had he attended the local high school, he would have been in the same class as future Mafia Boss John Gotti! Fortunately, Stan's scores on the citywide aptitude test placed him at the prestigious Stuyvesant High School, which has produced many notable scientists. Not the least among them was Peter Lax, who attended a decade earlier. Growing up in Brooklyn in the 1950's, Stan's main interests were (being Stan) science and (being Brooklyn) baseball. Indeed, his childhood measure of success was to earn as much as a professional baseball player. Thus he is a success, since the average 1950 player salary, corrected for 50 years of inflation, agrees with his current UCLA salary. On the other hand, UCLA does not match the current baseball player's \$2,000,000 average salary.

In 1958 Stan entered the Brooklyn College of the City University of New York, mainly due to the economic necessity of living at home. The same necessity lead many other future prominent mathematicians to attend there as well, including Jack Schwartz, Martin Davis and Paul Cohen. Stan majored in physics and went on to graduate study in math at Courant Institute in 1962. In doing so, he followed a path defined by his older sister, Sondra Osher-Jaffe, six years earlier. She was one of the remarkable and rare women to obtain a PhD in math at that time, preceding Stan at Brooklyn College and Courant where she, respectively, dated

Paul Cohen (once) and obtained a PhD in pseudo-analytic function theory under Lipman Bers. In 1962 Sondra was beginning her career as a professor at City College of New York (where she remained through 1998) and she assisted Stan in obtaining an NSF graduate fellowship for study in applied math at Courant. This paid all tuition plus \$3400 per year, tax free, which was quite a princely sum, or, as Stan says, "I've never lived so well since." Both Oshers saw math as an enjoyable path to a middle class life, and—at the peak of the Sputnik-era Cold War—also as a patriotic endeavor to contribute to U.S. scientific superiority.

Courant Institute in the early 1960's was home to such luminaries of post-World War II applied math as Friedrichs, Stoker, Fritz John, and Courant himself, as well as their eventual intellectual successors, including the young Peter Lax, Joe Keller and Heinz Kreiss. However, Stan originally had a pure math orientation, studying functional analysis and integral operators under Jacob Schwartz, and graduating in 1966 with a PhD Thesis entitled *Similarity Properties of Certain Volterra Operators on  $L^p([0, 1])$* . He spent a post-doctoral year at Brookhaven National Lab, where he began thinking about numerical analysis, and wrote his first and last computer programs. This was followed by a position as an assistant professor at Berkeley for 1968–70, a distinction he shared with mathematician and future Unabomber Theodore Kazinski. Stan settled into a faculty position at SUNY Stony Brook that spanned 1970–76, punctuated by frequent sabbaticals elsewhere, including a year in Sweden visiting Kreiss and a return to Courant for 1973 as a Sloan Fellow, where he began his collaborations with Andy Majda. Andy recruited Stan to UCLA in 1976, where he has remained since, despite the best efforts of several other institutions.

Stan credits two Courant colleagues with directing him into applied math and numerical methods for PDEs. The first was his postdoctoral office mate at Courant, Alexander Chorin, who suggested Stan apply his integral operator skills to questions outlined by Gilbert Strang in a 1966 paper on the stability of initial-boundary value problems. Stan devised a Toeplitz matrix-based formulation of the stability conditions, and the result was his first work on discretization methods. The second influential figure was Andy Majda, who Stan credits for simply impressing upon him that "it's good to work on problems people care about".

With his redirection into applied and computational math thus established, Stan was working on numerical methods for solving the full potential equation for transonic flow in the late 1970's at UCLA, with colleague Bjorn Engquist. This was of great interest at the time for computing steady flow past supersonic aircraft. Stan particularly focused on conditions and schemes that gave entropy-satisfying shock solutions. This work progressed to numerical scheme design for general gas dynamics, which in turn led to collaboration with the late Ami Harten, whom Stan had first met when he was a post-doc and Ami a graduate student at Courant. Ami had a grand vision of developing high accuracy numerical methods for general hyperbolic conservation laws, and their collaboration culminated in the development of the extremely successful "Essentially Non-Oscillatory (ENO)" family of shock-capturing schemes in the mid 1980's.

Stan's work on numerical methods for shocks and entropy conditions lead to a collaboration with Jamie Sethian, who as a graduate student of Chorin had been

studying the formation of kinks and intersections in flame fronts in model combustion problems. Jamie was interested in connecting these concepts with Stan's numerical schemes. Their efforts to develop such a numerical method during 1985–87 led them to create the general and powerful Level Set Method for computing interface motion. Stan credits reading a level set formulation of a theoretical problem in a 1985 report of Guy Barles for flipping the final mental switch needed to complete their numerical development effort.

Similarly, Stan's work on numerical methods for PDEs describing shocks and contact discontinuities caught the attention of image processing scientist Leonid Rudin, whom Stan had met while consulting at Rockwell in 1986. Leonid had noted in his recent CalTech PhD thesis that image processing manipulations could be viewed as applying evolution PDEs to the initial image data, and that the desired clean, sharp image edges must arise as shocks and contacts in the flow. This connection with hyperbolic conservation laws lead them, along with Stan's student Emad Fatemi, to develop PDE-based image processing via Total Variation (TV) Minimization methods during 1986–89. This breakthrough initiated a new approach to image processing, and also provided the technological basis for Cognitech, Inc., the image processing company Stan and Leonid co-founded. Cognitech is still in existence and expanding more than a decade later.

In the 1990's, Stan continued to surf the tidal waves unleashed by these three innovations, developing many extensions of ENO, PDE based image processing, and the Level Set Method, as well as new applications into areas such as Materials Science, Multiphase Flow, Photolithography, Data Compression and Computer Graphics. Stan also founded his second company, Level Set Systems, to commercialize the Level Set technology. And the rest is History—though Stan continues to actively generate new history, with undiminished energy and enthusiasm.

No account of Stan can do justice without conveying a little of his unique persona. Stan is a cross between Johnny Von Neumann and Woody Allen—lightening fast with the formulas, but speaking entirely in hilarious one-liners. He has many classic lines, such as the relation between front tracking and level set methods: “they're like capitalism and communism—they don't mix!” Or that the Level Set Method captures interface merging “with no emotional involvement from the programmer”. My first exposure to Stan was his invitation to a postdoctoral position at UCLA. I was at the University of Chicago, which was in the midst of a particularly frigid winter, and Stan phoned to say “come on out, you'll love it—we have the best computational math in the country, and there's naked women on roller skates!” Now, having spent a decade at UCLA and also living near Stan's former “bachelor pad” at Venice Beach, I can see he was right on both accounts.

Leaving Chicago, I was told that Osher was mainly an “empire builder”, as well as an exemplar of the California playboy lifestyle, with his shirt unbuttoned to his waist and gold chains around his neck. Stan must have shed the disco attire by the time I arrived in 1989, and I also learned firsthand that “empire builder” was something of a misrepresentation. Stan did indeed play a key role in building a computational math empire at UCLA, ultimately enabling it to attract prominent faculty such as Heinz Kreiss, and impressive facilities such as the NSF Institute for Pure and Applied Math. But Stan's day-to-day activities and interests centered

almost entirely around mathematical research. His office door was always open, Stan usually at his desk excitedly working out some new governing PDE or discretization. A new idea was always welcome, to be greeted by "That's Fantastic! Really incredible! But—Aha!—check this out, man..." and Stan would proceed to scrawl a few undecipherable lines on his blackboard, giving just the right formula or discretization. For the subsequent deciphering, Stan's long-time assistant Babette Dalton is the Rosetta Stone, being uniquely capable of translating Osher-script into text.

Even in his empire-builder mode, Stan maintained a true researcher's perspective. I recall him engaged in the planning of a large proposal, with his colleague outlining the major computer hardware requirements, when he interrupted "don't worry about equipment—it all depends on getting good people!" Indeed, Stan was always true to that theme, bringing in a steady stream of innovative and diverse mathematicians, plus scientists from other disciplines bearing novel problems. Stan would hold court, and we would gather and listen to new techniques or applications. But midway through the discussions, Stan was always quick to cut to the point "...all right, all right—but what's the equation?!" Given a PDE, Stan would correct any ill-posedness, scratch out a quick non-oscillatory discretization, and dispatch a graduate student or post-doc to program the resulting scheme. Results were expected by the next day—if not sooner—but Stan always used the carrot of his enthusiasm, rather than the stick, to drive this pace. This was the Osher *modus operandi*, and it continues to produce a high level of adrenaline and discovery in his applied math group at UCLA.

It is certain this introduction has gone on too long for Stan's taste. By this point, he would cut it off with his characteristic "all right, all right—but what's the equation?!" Okay, it's simple: what's the equation? Osher = 60!

*Barry Merriman*  
*University of California, Los Angeles*  
*March 2003*

The volume is from the proceedings of the international conference held in celebration of Stanley Osher's sixtieth birthday. It presents recent developments and exciting new directions in scientific computing and partial differential equations for time dependent problems and its interplay with other fields, such as image processing, computer vision and graphics. Over the past decade, there have been very rapid developments in the field. This volume emphasizes the strong interaction of advanced mathematics with real-world applications and algorithms.

The book is suitable for graduate students and research mathematicians interested in scientific computing and partial differential equations.

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