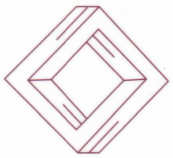
CONTEMPORARY MATHEMATICS

340

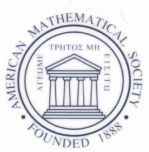


APORTACIONES MATEMÁTICAS
SOCIEDAD MATEMÁTICA MEXICANA

Spectral Theory of Schrödinger Operators

Lecture Notes from a Workshop on
Schrödinger Operator Theory
December 3 – 7, 2001
Universidad Nacional Autónoma de México,
IIMAS, Mexico

Rafael del Río
Carlos Villegas-Blas
Editors



American Mathematical Society

http://dx.doi.org/10.1090/conm/340

Spectral Theory of Schrödinger Operators

CONTEMPORARY MATHEMATICS

340

APORTACIONES MATEMÁTICAS
SOCIEDAD MATEMÁTICA MEXICANA

Spectral Theory of Schrödinger Operators

Lecture Notes from a Workshop on
Schrödinger Operator Theory
December 3 - 7, 2001
Universidad Nacional Autónoma de México,
IIMAS, Mexico

Rafael del Río
Carlos Villegas-Blas
Editors

American Mathematical Society
Providence, Rhode Island

This volume contains the Lecture Notes from A Workshop on Schrödinger Operator Theory that was held at The Institute of Applied Mathematics (IIMAS) of the National University of Mexico (UNAM) from December 3–7, 2001. The workshop was partially supported by Proyect UNAM–PAPIIT IN-102998 and by funds provided by IIMAS.

2000 *Mathematics Subject Classification.* Primary 81Q10, 81–06, 81Q05; Secondary 81–02.

Library of Congress Cataloging-in-Publication Data

Spectral theory of Schrödinger operators / Rafael del Río, Carlos Villegas-Blas, editors.

"Lecture notes from a workshop on Schrödinger operator theory, December 3–7, 2001, Universidad Nacional Autónoma de México, IIMAS, Mexico."

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 340) (Aportaciones Matemáticas)

Includes bibliographical references.

ISBN 0-8218-3297-2 (alk. paper)

1. Schrödinger operator—Congresses I. Río, Rafael del, 1957– II. Villegas-Blas, Carlos, 1964– III. Universidad Nacional Autónoma de México. Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas. IV. Series. V. Contemporary mathematics (American Mathematical Society); v. 340.

QC174.17.S3S64 2004
515′.724—dc22 2003063783

Contents

PREFACE

During the first week of December 2001, we organized a workshop on Schrödinger Operators at The Institute of Applied Mathematics (IIMAS) of the National University of Mexico (UNAM). We were very fortunate to have the participation of outstanding mathematicians who delivered a series of lectures on different topics concerning Schrödinger Operator Theory. This publication contains the lecture notes of the workshop.

The main purpose of the meeting was to offer a series of courses with the goal to provide a non-specialized audience with basic tools to understand and appreciate new trends of recent research. Thus the speakers were requested to give a survey of their area of work, to introduce the necessary mathematical background and then to link these with present research on the subject including their own work. We believe the Lectures Notes appearing in this volume reflect faithfully the essential idea of the workshop. The authors of the Lecture Notes are: Francois Bentosela, Johannes Brasche, Andreas M. Hinz, Pavel Kurasov and Ivan Veselić.

The study of selfadjoint operators in Hilbert Spaces, particularly of Schrödinger operators, has played a role of great importance in the developement of the mathematical theory of Quantum Mechanics. Since the last century, an impressive amount of work has been done in several related areas, such as Spectral Theory, Scattering Theory, Ordinary and Partial Differential Equations, Functional Analysis and, more recently, Random Schrödinger Operators and the study of particular types of Spectra, like dense point and singular continuous. It is for this reason that surveys like the ones presented in these Lecture Notes should be extremely useful as a guide for the mathematician interesed in entering into these areas.

The order in which the papers are presented in this volume is intended to provide the reader first with the papers whose content might help to understand the others. However, each paper is self-contained and the reader can choose any one of the papers to read independently of the others.

The paper by A. Hinz surveys several aspects of spectral theory of differential operators. It gives a modern presentation of the theory from the very beginnings to topics of current research. In particular, it is shown how the problem of uniqueness and existence of solutions can be reduced to the problem of establishing self-adjointness of a linear operator. Schrödinger Operators, which are radially periodic, are studied and the author includes his own research work.

The paper by J. Brasche has standard material on self-adjoint operators: resolution of the identity, the functional calculus, and decomposition of a self-adjoint operator into its continuous, absolutely continuous and discrete parts. He discusses Kato-Rosenblum, the so-called RAGE theorem, i.e., the identification of eigenstates

with "bound" states, and scattering states with the continuous part of the operator. There is a discussion of Schatten classes of compact operators, perturbation of self-adjoint operators by compact operators, and the stability of the spectrum under these perturbations. The article includes a discussion of symmetric operators and their self-adjoint extensions (if any) and what the spectral theory of these extensions can look like, in particular in spectral gaps of the original operator. The study of the Laplacian Δ defined on $C^\infty(R^n - \Gamma)$ and acting in $L^2(R^n)$, with Γ a set of measure zero, is such an example. He introduces the reader to the notion of capacity, Hedberg's theorem and the questions of extensions of Δ.

The paper by I. Veselić reviews spectral properties of random Schrödinger Operators. The focus is set on the integrated density of states (IDS). It includes an exhaustive discussion of recent results on Wegner estimates and their importance for localization and it contains a quite complete list of references.

The paper by P. Kurasov is about finite rank singular perturbations of positive self-adjoint operators. These kind of perturbations lead to models of complex quantum systems which are exactly solvable. It contains a discussion of singular and supersingular perturbations. Many recent developments by the author are included.

F. Bentosela studies the scattering theory of Schrödinger Operators (both continuous and discrete) when we have a surface potential. He studies two models where he gives the generalized eigenfunctions and discusses spectral properties.

This is a joint publication of the American and Mexican Mathematical Societies. We wish that the fruitful collaboration between both Societies will last very long.

Acknowledgements

The workshop was supported partially by Proyect UNAM-PAPIIT IN-102998 and by funds provided by IIMAS. We are very grateful to the authors and the referees for their collaboration and to the staff of the AMS for their help.

The Editors
Carlos Villegas-Blas[1] and Rafael del Río[2] .

Universidad Nacional Autónoma de México
Instituto de Matemáticas
Unidad Cuernavaca
A. P. 273-3, Admon. 3,
Cuernavaca Morelos 62251
México

Universidad Nacional Autónoma de México
IIMAS
Apdo. Postal 20-726, Admon. 20
Delegación Álvaro Obregón,
México, Distrito Federal, 01000
México

[1]e-mail: villegas@matcuer.unam.mx
[2]e-mail: delrio@leibniz.iimas.unam.mx

Titles in This Series

341 **Hugo Arizmendi, Carlos Bosch, and Lourdes Palacios, Editors,** Topological algebras and their applications, 2004

340 **Rafael del Río and Carlos Villegas-Blas, Editors,** Spectral theory of Schrödinger operators, 2004

339 **Peter Kuchment, Editor,** Waves in periodic and random media, 2003

338 **Pascal Auscher, Thierry Coulhon, and Alexander Grigor'yan, Editors,** Heat kernels and analysis on manifolds, graphs, and metric spaces, 2003

337 **Krishan L. Duggal and Ramesh Sharma, Editors,** Recent advances in Riemannian and Lorentzian geometries, 2003

336 **José González-Barrios, Jorge A. León, and Ana Meda, Editors,** Stochastic models, 2003

335 **Geoffrey L. Price, B. Mitchell Baker, Palle E.T. Jorgensen, and Paul S. Muhly, Editors,** Advances in quantum dynamics, 2003

334 **Ron Goldman and Rimvydas Krasauskas, Editors,** Topics in algebraic geometry and geometric modeling, 2003

333 **Giovanni Alessandrini and Gunther Uhlmann, Editors,** Inverse problems: Theory and applications, 2003

332 **John Bland, Kang-Tae Kim, and Steven G. Krantz, Editors,** Explorations in complex and Riemannian geometry, 2003

331 **Luchezar L. Avramov, Marc Chardin, Marcel Morales, and Claudia Polini, Editors,** Commutative algebra: Interactions with algebraic geometry, 2003

330 **S. Y. Cheng, C.-W. Shu, and T. Tang, Editors,** Recent advances in scientific computing and partial differential equations, 2003

329 **Zhangxin Chen, Roland Glowinski, and Kaitai Li, Editors,** Current trends in scientific computing, 2003

328 **Krzysztof Jarosz, Editor,** Function spaces, 2003

327 **Yulia Karpeshina, Günter Stolz, Rudi Weikard, and Yanni Zeng, Editors,** Advances in differential equations and mathematical physics, 2003

326 **Kenneth D. T-R McLaughlin and Xin Zhou, Editors,** Recent developments in integrable systems and Riemann-Hilbert problems, 2003

325 **Seok-Jin Kang and Kyu-Hwan Lee, Editors,** Combinatorial and geometric representation theory, 2003

324 **Caroline Grant Melles, Jean-Paul Brasselet, Gary Kennedy, Kristin Lauter, and Lee McEwan, Editors,** Topics in algebraic and noncommutative geometry, 2003

323 **Vadim Olshevsky, Editor,** Fast algorithms for structured matrices: theory and applications, 2003

322 **S. Dale Cutkosky, Dan Edidin, Zhenbo Qin, and Qi Zhang, Editors,** Vector bundles and representation theory, 2003

321 **Anna Kamińska, Editor,** Trends in Banach spaces and operator theory, 2003

320 **William Beckner, Alexander Nagel, Andreas Seeger, and Hart F. Smith, Editors,** Harmonic analysis at Mount Holyoke, 2003

319 **W. H. Schikhof, C. Perez-Garcia, and A. Escassut, Editors,** Ultrametric functional analysis, 2003

318 **David E. Radford, Fernando J. O. Souza, and David N. Yetter, Editors,** Diagrammatic morphisms and applications, 2003

317 **Hui-Hsiung Kuo and Ambar N. Sengupta, Editors,** Finite and infinite dimensional analysis in honor of Leonard Gross, 2003

316 **O. Cornea, G. Lupton, J. Oprea, and D. Tanré, Editors,** Lusternik-Schnirelmann category and related topics, 2002

315 **Theodore Voronov, Editor,** Quantization, Poisson brackets and beyond, 2002

This volume gathers the articles based on a series of lectures from a workshop held at the Institute of Applied Mathematics of the National University of Mexico. The aim of the book is to present to a non-specialized audience the basic tools needed to understand and appreciate new trends of research on Schrödinger operator theory.

Topics discussed include various aspects of the spectral theory of differential operators, the theory of self-adjoint operators, finite rank perturbations, spectral properties of random Schrödinger operators, and scattering theory for Schrödinger operators.

The material is suitable for graduate students and research mathematicians interested in differential operators, in particular, spectral theory of Schrödinger operators.

ISBN 0-8218-3297-2

9 780821 832974

CONM/340

American Mathematical Society
www.ams.org

Sociedad Matemática Mexicana
www.smm.org.mx