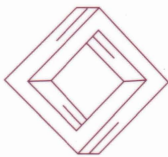


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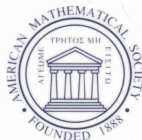


APORTACIONES MATEMÁTICAS
SOCIEDAD MATEMÁTICA MEXICANA

Spectral Theory of Schrödinger Operators

Lecture Notes from a Workshop on
Schrödinger Operator Theory
December 3 – 7, 2001
Universidad Nacional Autónoma de México,
IIMAS, Mexico

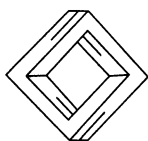
Rafael del Río
Carlos Villegas-Blas
Editors



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PREFACE

During the first week of December 2001, we organized a workshop on Schrödinger Operators at The Institute of Applied Mathematics (IIMAS) of the National University of Mexico (UNAM). We were very fortunate to have the participation of outstanding mathematicians who delivered a series of lectures on different topics concerning Schrödinger Operator Theory. This publication contains the lecture notes of the workshop.

The main purpose of the meeting was to offer a series of courses with the goal to provide a non-specialized audience with basic tools to understand and appreciate new trends of recent research. Thus the speakers were requested to give a survey of their area of work, to introduce the necessary mathematical background and then to link these with present research on the subject including their own work. We believe the Lecture Notes appearing in this volume reflect faithfully the essential idea of the workshop. The authors of the Lecture Notes are: Francois Bentosela, Johannes Brasche, Andreas M. Hinz, Pavel Kurasov and Ivan Veselić.

The study of selfadjoint operators in Hilbert Spaces, particularly of Schrödinger operators, has played a role of great importance in the development of the mathematical theory of Quantum Mechanics. Since the last century, an impressive amount of work has been done in several related areas, such as Spectral Theory, Scattering Theory, Ordinary and Partial Differential Equations, Functional Analysis and, more recently, Random Schrödinger Operators and the study of particular types of Spectra, like dense point and singular continuous. It is for this reason that surveys like the ones presented in these Lecture Notes should be extremely useful as a guide for the mathematician interested in entering into these areas.

The order in which the papers are presented in this volume is intended to provide the reader first with the papers whose content might help to understand the others. However, each paper is self-contained and the reader can choose any one of the papers to read independently of the others.

The paper by A. Hinz surveys several aspects of spectral theory of differential operators. It gives a modern presentation of the theory from the very beginnings to topics of current research. In particular, it is shown how the problem of uniqueness and existence of solutions can be reduced to the problem of establishing self-adjointness of a linear operator. Schrödinger Operators, which are radially periodic, are studied and the author includes his own research work.

The paper by J. Brasche has standard material on self-adjoint operators: resolution of the identity, the functional calculus, and decomposition of a self-adjoint operator into its continuous, absolutely continuous and discrete parts. He discusses Kato-Rosenblum, the so-called RAGE theorem, i.e., the identification of eigenstates

with “bound” states, and scattering states with the continuous part of the operator. There is a discussion of Schatten classes of compact operators, perturbation of self-adjoint operators by compact operators, and the stability of the spectrum under these perturbations. The article includes a discussion of symmetric operators and their self-adjoint extensions (if any) and what the spectral theory of these extensions can look like, in particular in spectral gaps of the original operator. The study of the Laplacian Δ defined on $C^\infty(R^n - \Gamma)$ and acting in $L^2(R^n)$, with Γ a set of measure zero, is such an example. He introduces the reader to the notion of capacity, Hedberg’s theorem and the questions of extensions of Δ .

The paper by I. Veselić reviews spectral properties of random Schrödinger Operators. The focus is set on the integrated density of states (IDS). It includes an exhaustive discussion of recent results on Wegner estimates and their importance for localization and it contains a quite complete list of references.

The paper by P. Kurasov is about finite rank singular perturbations of positive self-adjoint operators. These kind of perturbations lead to models of complex quantum systems which are exactly solvable. It contains a discussion of singular and supersingular perturbations. Many recent developments by the author are included.

F. Bentosela studies the scattering theory of Schrödinger Operators (both continuous and discrete) when we have a surface potential. He studies two models where he gives the generalized eigenfunctions and discusses spectral properties.

This is a joint publication of the American and Mexican Mathematical Societies. We wish that the fruitful collaboration between both Societies will last very long.

Acknowledgements

The workshop was supported partially by Proyect UNAM-PAPIIT IN-102998 and by funds provided by IIMAS. We are very grateful to the authors and the referees for their collaboration and to the staff of the AMS for their help.

The Editors

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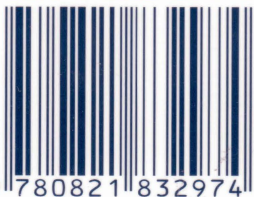
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This volume gathers the articles based on a series of lectures from a workshop held at the Institute of Applied Mathematics of the National University of Mexico. The aim of the book is to present to a non-specialized audience the basic tools needed to understand and appreciate new trends of research on Schrödinger operator theory.

Topics discussed include various aspects of the spectral theory of differential operators, the theory of self-adjoint operators, finite rank perturbations, spectral properties of random Schrödinger operators, and scattering theory for Schrödinger operators.

The material is suitable for graduate students and research mathematicians interested in differential operators, in particular, spectral theory of Schrödinger operators.

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