

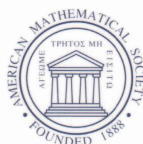
CONTEMPORARY MATHEMATICS

358

Stark's Conjectures: Recent Work and New Directions

An International Conference
on Stark's Conjectures and Related Topics
August 5–9, 2002
Johns Hopkins University

David Burns
Cristian Popescu
Jonathan Sands
David Solomon
Editors



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Dedicated to Harold Stark and John Tate

Contents

Preface	vii
List of Participants	ix
Rubin's integral refinement of the abelian Stark conjecture CRISTIAN D. POPESCU	1
Computations related to Stark's conjecture D. S. DUMMIT	37
Arithmetic annihilators and Stark-type conjectures CORNELIUS GREITHER	55
The equivariant Tamagawa number conjecture: A survey MATTHIAS FLACH	79
Popescu's conjecture in multi-quadratic extensions JONATHAN W. SANDS	127
Abelian conjectures of Stark type in \mathbb{Z}_p -extensions of totally real fields D. SOLOMON	143
The derivative of p-adic Dirichlet series at $s = 0$ H. M. STARK	179
Refining Gross's conjecture on the values of abelian L -functions JOHN TATE	189
Stickelberger functions for non-abelian Galois extensions of global fields DAVID R. HAYES	193
Introduction to Kolyvagin systems BARRY MAZUR and KARL RUBIN	207

Preface

Between 1971 and 1980, Harold Stark published a series of four papers in *Advances in Mathematics* concerning the Artin L -functions $L(s, \chi)$ associated to a Galois extension of number fields K/k . In the general case, he conjectured that their leading terms at $s = 0$ (or, by the functional equation, at $s = 1$) should be equal to a certain ‘ χ -regulator’ of S -units of K times an algebraic constant depending $\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ -equivariantly on χ . Under special conditions he made conjectures with stronger consequences, e.g. a solution of Hilbert’s 12th Problem in certain instances where the conjecture is ‘first order’.

These conjectures were rapidly taken up and developed by other number theorists, including Chinburg, Deligne, Gross, Hayes, Serre, and Tate. The last-named also gave a course on the subject at Orsay in 1980/81. Lecture notes written up by Bernardi and Schappacher gave rise to Tate’s 1984 book, *Les conjectures de Stark sur les fonctions L d’Artin en $s = 0$* , which collected together most of the developments up to that date. In the years that followed, this became the standard reference for researchers on Stark’s Conjectures. Steady progress was made in the area alongside – and often interacting with – advances on certain other ‘ L -function conjectures’ then being actively investigated (to cite but three: those due to Bloch and Kato and to Fontaine and Perrin-Riou, as well as the Main Conjecture of Iwasawa Theory). By the turn of the millennium, there was a clear need to take stock of the ‘state-of-the-art’ in the area. With this in mind, the editors organised a conference on *Stark’s Conjectures and Related Topics*, held at Johns Hopkins University in Baltimore from 5 to 9 August 2002. The more than 60 participants included almost all the experts in the field. At the conclusion of a very successful week there was general enthusiasm for creating a permanent record of its proceedings.

This volume is the result. It does not claim to replace Tate’s book as an introduction to the Stark Conjectures. Nor, coming almost exactly twenty years later, can it pretend to catalogue all the intervening work in the area. The ten articles which it contains cannot even cover the great thematic diversity of the more than thirty talks given at the conference. Nevertheless, almost all of the authors were among the invited speakers, and most of their contributions expand on the subject matter of their plenary talks. It is therefore the editors’ belief that the first four, ‘survey-type’ contributions provide an introduction to a good part of the recent work relating to Stark’s Conjectures. The remaining six should give a taste of some major themes currently being explored in the area (non-abelian and p -adic aspects of the conjectures, abelian refinements, etc.). At the same time, we are acutely conscious of some major omissions, and of two in particular: we have been unable to include Gross’s p -adic refinement of the ‘first order’ abelian Stark Conjecture and its recent generalizations to the ‘arbitrary order’ abelian Stark

Conjecture. The reader will also have to look elsewhere for an account of Burns's recent important work explicitly linking Stark's Conjectures to the Equivariant Tamagawa Number Conjecture.

The repetition of the word 'conjecture' in the above paragraphs – and throughout the volume – reminds one that this is an area where important achievements and insights can seem overshadowed by the amount that still remains to be *proven*. We trust that, far from despair, this contrast will be cause for excitement, inspiring the further exploration of a vision whose pursuit has already revealed so much deep and beautiful number theory.

We owe an enormous debt of gratitude to the conference participants for their enthusiastic contributions and the wonderful atmosphere that they helped create. Thanks are due also to Johns Hopkins University and its helpful staff, and the National Science Foundation and the Number Theory Foundation for supporting the conference both financially and practically. Many thanks, finally, to the staff of the AMS for their help and guidance during the lengthy process of producing this volume.

The Editors, 17 February 2004

List of Participants

A. Agboola University of California, Santa Barbara	M. Flach California Institute of Technology
J.-R. Belliard Université de Besançon	K. Fogel California Lutheran University
W. Bley Universität Augsburg	S. Ganguly Rutgers University
M. Breuning King's College, London	D. Grant University of Colorado
A. Brumer Fordham University	C. Greither Universität der Bundeswehr München
D. Burns King's College, London,	B. Gross Harvard University
B. Cha Johns Hopkins University	P. Gunnells University of Massachusetts, Amherst
P. Charollois Université Bordeaux I	W. Hart Macquarie University
J. Colwell California Institute of Technology	D. Hayes University of Massachusetts, Amherst
B. Conrad University of Michigan	A. Hayward King's College, London
A. Cooper King's College, London	J. Hooper Acadia University
H. Darmon McGill University	B. Howard Harvard University
S. Dasgupta University of California, Berkeley	D. Hubbard U.S. Department of Defense
E. de Shalit Hebrew University of Jerusalem	B. Kim Stanford University
D. Dummit University of Vermont	J. Klefe Hong Kong

M. Kolster
McMaster University

M. Kurihara
Tokyo Metropolitan University

J. Lee
Korean Institute for Advanced Study

Q. Lin
California Institute of Technology

M. Marcolli
Universität Bonn

T. Nguyen Quang Do
Université de Besançon

T. Ono
Johns Hopkins University

C. Popescu
Johns Hopkins University

K. Prasana
Princeton University

N. Ramachandran
University of Maryland

M. Reid
University of Arizona

X. Roblot
Université Lyon I

K. Rubin
Stanford University

J. Sands
University of Vermont

R. Sczech
Rutgers University, Newark

R. Sharifi
Harvard University

V. Snaith
University of Southampton

D. Solomon
King's College, London

H. Stark
University of California, San Diego

K.-S. Tan
National Taiwan University

B. Tangedal
University of Charleston

J. Tate
University of Texas

R. Torrey
University of Vermont

M. Volpato
Macquarie University

L. Washington
University of Maryland

A. Weiss
University of Alberta

S.-W. Zhang
Columbia University

Stark's conjectures on the behavior of L -functions were formulated in the 1970s. Since then, these conjectures and their generalizations have been actively investigated. This has led to significant progress in algebraic number theory.

The current volume, based on the conference held at Johns Hopkins University (Baltimore, MD), represents the state-of-the-art research in this area. The first four survey papers provide an introduction to a majority of the recent work related to Stark's conjectures. The remaining six contributions touch on some major themes currently under exploration in the area, such as non-abelian and p -adic aspects of the conjectures, abelian refinements, etc. Among others, some important contributors to the volume include Harold M. Stark, John Tate, and Barry Mazur.

The book is suitable for graduate students and researchers interested in number theory.

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