

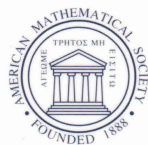
CONTEMPORARY MATHEMATICS

377

Idempotent Mathematics and Mathematical Physics

International Workshop
February 3–10, 2003
Erwin Schrödinger International Institute
for Mathematical Physics
Vienna, Austria

G. L. Litvinov
V. P. Maslov
Editors



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Preface

Idempotent mathematics is a new branch of mathematical sciences, rapidly developing and gaining popularity over the last decade; it is also closely related to mathematical physics. The literature on the subject is vast and includes numerous books and an all but innumerable body of journal papers; a wide sample of references can be found in G. L. Litvinov's introductory paper and other papers published in this volume.

An important stage of development of the subject was presented in the book *Idempotency* edited by J. Gunawardena (Publ. of the Newton Institute, Vol. 11, Cambridge University Press, Cambridge, 1998). This book arose out of the well-known workshop that was held in Bristol, England, in October 1994.

To snapshot modern idempotent mathematics at a new stage of its development, we have organized a workshop on *Idempotent Mathematics and Mathematical Physics* hosted by the Erwin Schrödinger Institute for Mathematical Physics in Vienna, Austria, in February 2003. The present volume provides an extended record of this meeting along with a number of invited contributions. We believe that the table of contents is self-explanatory.

It is a pleasure to thank the Erwin Schrödinger Institute for Mathematical Physics, American Mathematical Society and Russian Fund for Basic Research (grant 02-01-01062) for their important support. We are grateful to Sergei Gelfand and Christine Thivierge of the American Mathematical Society, to Peter Michor, Klaus Schmidt, and Maria Windhager of the Erwin Schrödinger Institute, as well as to all the officers of this Institute, and to a number of colleagues, especially to Andreï Sobolevskiï of the Moscow State University, for their great help. We thank all the authors of this volume and members of our "idempotent/max-plus/tropical community" for their contributions, help, and useful contacts.

G. L. Litvinov and V. P. Maslov
Moscow, December 2004

Idempotent mathematics is a rapidly developing new branch of the mathematical sciences that is closely related to mathematical physics. The existing literature on the subject is vast and includes numerous books and journal papers.

A workshop was organized at the Erwin Schrödinger Institute for Mathematical Physics (Vienna) to give a snapshot of modern idempotent mathematics. This volume contains articles stemming from that event. Also included is an introductory paper by G. Litvinov and additional invited contributions.

The resulting volume presents a comprehensive overview of the state of the art. It is suitable for graduate students and researchers interested in idempotent mathematics and tropical mathematics.

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