CONTEMPORARY MATHEMATICS

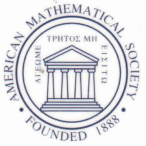
396

# Complex Dynamics

## Twenty-Five Years after the Appearance of the Mandelbrot Set

Proceedings of an AMS-IMS-SIAM
Joint Summer Research Conference
on Complex Dynamics: Twenty-Five Years
after the Appearance of the Mandelbrot Set
June 13–17, 2004
Snowbird, Utah

Robert L. Devaney
Linda Keen
Editors




American Mathematical Society

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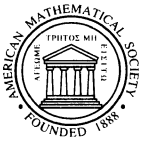


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# Preface

The field of complex dynamical systems has undergone two periods of remarkable growth. The first occurred in the early twentieth century. During this period, much of the basic behavior of complex analytic dynamical systems was described in numerous works of Gaston Julia, Pierre Fatou, and their contemporaries. Earlier work in complex dynamics by Leau, Koenigs, and Böttcher, among others, had focused on the question of the linearizability of an analytic function in the neighborhood of a fixed or periodic point. In a domain where such a linearization is possible, the map in question is analytically conjugate to a linear map, so the dynamics are perfectly well understood and "stable" in some sense. The work on linearization questions quickly came to a halt when the question of the linearizability near a fixed point whose derivative was an irrational rotation arose. So Julia and Fatou branched out in a different direction: they worked mainly on iteration of a complex map in the boundaries of the domains where linearizability occurs, i.e., the set of points now known as the Julia set. Using the recently proved theorem of Montel, Julia and Fatou were able to develop many of the properties of these sets. For example, they knew that many of these sets were fractal in nature, although of course the term "fractal" was not in use at the time. Similarly, they could show that the map in question had rich, chaotic behavior on the Julia set, though again the term "chaos" had not yet been used in mathematics. Curiously, Julia also knew about the "fundamental dichotomy" in the dynamics of quadratic polynomials: that the Julia sets of these maps were either connected or totally disconnected and that the fate of the orbit of the only critical point determined which case occurred. Again, curiously, Julia never looked at (at least to the best of anyone's knowledge) the set of parameters for which the connected Julia sets occur, the set we now know as the Mandelbrot set. Of course, the lack of computers might have had some impact on this. Nevertheless, this area of research also ground to a halt, due primarily to the fact that nobody could come up with a complete classification of the stable domains, that is, the set where the dynamics was well-understood or the complement of the Julia set. The possibility of the existence of irrational rotation domains remained one headache, but the existence of wandering domains could also not be ruled out. So complex dynamics went to sleep for a half-century.

The field experienced a remarkable reawakening twenty five years ago. For it was then that the first good computer pictures of the Mandelbrot set and Julia sets appeared. These intriguing images seduced mathematicians from many different research areas to reinvestigate the dynamics of complex analytic functions. During the intervening years since the period of Julia and Fatou, numerous new techniques in mathematics had been developed, and this allowed researchers to overcome many of the hurdles encountered during the Julia-Fatou era.

In the early sixties, Ahlfors and Bers proved their "Measurable Riemann mapping theorem" which enabled them to use quasi-conformal mappings and what are now called holomorphic motions to study finitely generated Kleinian groups which are also complex analytic dynamical systems. Sullivan introduced a dictionary to translate between the dynamical systems arising from polynomials and those that arise from Kleinian groups that depends heavily on holomorphic motions. He used it to give a proof of the Ahlfors' finiteness conjecture for Kleinian groups, that the quotient of the Fatou set by the group has finitely many components. This removed one of the main obstacles dating from the original work on complex dynamics, namely the possibility of the existence of wandering domains for rational maps.

Thurston also used holomorphic motions to study how combinatorial data determine rational maps and Douady and Hubbard used them to show how phenomena in the dynamical plane and parameter plane are related. They also developed the method of external rays to explain almost all of the features of the Mandelbrot set. Finally, Yoccoz, building on earlier work of Siegel and Brjuno, settled many of the questions revolving about the existence of irrational rotation domains.

In the twenty five years since the Mandelbrot set was first seen, many other developments have occurred in the field of complex dynamics. At first, most of the research concentrated on the dynamics of polynomial and rational functions. More recently, many new areas of investigation have developed. Beyond rational maps, now researchers look into the behavior of entire and meromorphic maps as well as higher dimensional analytic maps (such as the Hénon map). Ergodic properties of the maps and the topological and geometric properties of the Julia sets have all become subfields in their own right. And much more remains to be done — for example, the question of the local connectivity of the boundary of the Mandelbrot set remains open. Once this question is resolved, many of the questions regarding the dynamics of quadratic polynomials will be put to rest. Beyond that, however, lies the much more complicated world of higher degree polynomials and analytic maps of other types. Hopefully, the stumbling block caused by local connectivity will soon be removed, allowing complex dynamicists to move into these more complicated realms with a sense that they too can be successfully analyzed.

This Proceedings is a report on the conference entitled Twenty Five Years After the Appearance of the Mandelbrot Set held at Snowbird, Utah, on June 13–17, 2004. The conference was organized by Bodil Branner, Eric Bedford, Mikhail Lyubich, and the co-editors. This conference was both a celebration of the remarkable resurgence of interest in complex dynamics spurred by the first pictures of the Mandelbrot set as well as a summary of the different directions now being pursued by researchers in this field.

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Chaotic behavior of (even the simplest) iterations of polynomial maps of the complex plane was known for almost one hundred years due to the pioneering work of Farou, Julia, and their contemporaries. However, it was only twenty-five years ago that the first computer generated images illustrating properties of iterations of quadratic maps appeared. These images of the so-called Mandelbrot and Julia sets immediately resulted in a strong resurgence of interest in complex dynamics. The present volume, based on the talks at the conference commemorating the twenty-fifth anniversary of the appearance of Mandelbrot sets, provides a panorama of current research in this truly fascinating area of mathematics.

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